Control Problems for Hyperbolic Equations

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The study of hyperbolic systems of of balance laws from the control theory point of view is receiving an increasing attention in the recent years both in the case of controls affecting the evolution of the system through the boundary conditions, and in the case of controls acting directly on the equation or on the source term. The interest on such problems is motivated by applications to traffic flow models, multicomponent chromatography, as well as in problems of oil resevoir simulation and gas dynamic.

In this talk, after discussing some results concerning the finite time exact controllability and the asymptotic stabilizzability, we shall address the problem of determining an optimal (entropy weak) solution relative to cost functionals with boundary or distributed controls. Here, the main source of difficulties stems from the fact that the "input-to-trajectory map" that associates to a given control the corresponding solution may not be differentiable in any natural Banach space. In this context, we shall discuss the use of a suitable differential structure compatible with the flow generated by conservation laws and how to implement it to derive a Pontryagin-type maximum principle for optimal controls.