Stabilized Residual Distribution Schemes for Shallow Water Simulations

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We develop a new conservative Residual Distribution scheme on unstructured triangular meshes for the solution of the two-dimensional Shallow Water Equations. Our starting point are the schemes proposed in [2, 4, 5]. These schemes are well balanced and essentially non oscillatory, and yet they have a very compact stencil. However, there have also been some open problems: the iterative solver suffers from convergence problems in the time-dependent case, leading to sub-optimal accuracy, and the schemes are not stable near dry zones. In this contribution we will present a cure for both problems.

To overcome the convergence issue for unsteady problems, we follow [1, 3] by adding a high order streamline dissipation term in smooth areas of the approximate solution. The treatment of dry regions requires a scheme independent of the flux Jacobians, which are fundamental for the upwind-type schemes used in [5]. In detail, we perform the following steps:

**Nonlinear convergence and distribution strategy** We need a flexible distribution strategy, allowing to handle both dry and flooded areas. To enlarge as much as possible the spectrum of possibilities, we adopted the stabilized formulation of limited nonlinear residual distribution proposed in [1] and [3].

**Choice and testing of a scheme** We analyzed and extensively tested discretizations based on a nonlinear variant of a multidimensional Lax-Friedrichs scheme. As we will show, this approach allows to build schemes that yield results as accurate as the ones obtained with the upwind schemes proposed in [5].

**Dry bed computations** Concerning the positivity of the water height, we will present minor modifications to the developed schemes in cells at the wetting-drying front. We will prove that these schemes keep the water height positive, without changing the properties in the flooded areas. The behaviour will be demonstrated on various test cases, including flooding and drying on different bottom topologies.

References


Figure 1: Wave runup behind a conical island
