Relaxation limits and asymptotic behaviors of solutions to the hydrodynamic model for semiconductors

Shinya Nishibata
Tokyo Institute of Technology, Japan
shinya@is.titech.ac.jp

Several kinds of models are proposed for analysis and device simulation on describing the electron flow through semiconductor devices. Especially the hydrodynamic, the energy transport and the drift-diffusion models are frequently utilized for the simulation with the suitable choice according to the purpose of use of real devices. Hence mathematical analysis on the solvability of these models globally in time and their model hierarchy are important problems not only in mathematics but also in engineering. The model hierarchy is formally understood by the limit procedure to make a momentum relaxation time τ_m and/or an energy relaxation time τ_e tend to zero. The main purpose of this talk is to discuss the solvability of the models and then justification of the relaxation limit procedures rigorously.

The hydrodynamic model is a system of equations

$$\rho_t + j_x = 0, \tag{H1}$$

$$j_t + \left(j^2/\rho + \theta\rho\right)_x = \rho\phi_x - j/\tau_m, \tag{H2}$$

$$\rho\theta_t + j\theta_x + \frac{2}{3}\left(\frac{j}{\rho}\right)_x \rho\theta - \frac{2}{3}\left(\kappa\theta_x\right)_x = \frac{2\tau_e - \tau_m}{3\tau_m\tau_e}\frac{j^2}{\rho} - \frac{\rho}{\tau_e}(\theta - \bar{\theta}), \tag{H3}$$

$$\phi_{xx} = \rho - D \tag{H4}$$

for a spatial variable $x \in \Omega := (0,1)$ and a time variable $t > 0$. Here electron density ρ, electric current j, absolute temperature θ and electrostatic potential ϕ are unknown functions. Positive constants $\bar{\theta}$, κ, τ_m and τ_e mean ambient device temperature, thermal conductivity, momentum relaxation time and energy relaxation time, respectively. Doping profile $D(x)$, which determines the electric property of semiconductors, is a positive and bounded continuous function of the spatial variable x. The initial and the boundary conditions to the system (H) are prescribed as

$$(\rho, j, \theta)(0,x) = (\rho_0, j_0, \theta_0)(x), \tag{I}$$

$$\rho(t,0) = \rho_l, \quad \rho(t,1) = \rho_r, \quad \theta_x(t,0) = \theta_x(t,1) = 0, \quad \phi(t,0) = 0, \quad \phi(t,1) = \phi_r, \tag{B}$$

where ρ_l, ρ_r and ϕ_r are given positive constants. Substituting $s := t/\tau_m$, $J := j/\tau_m$ and $\kappa_0 := \kappa/\tau_m$, in (H) yields the system

$$\rho_s + J_x = 0, \tag{H'1}$$

$$\tau_m^2 J_s + \left(\tau_m^2 J^2/\rho + \rho\theta\right)_x = \rho\phi_x - J, \tag{H'2}$$

$$\rho\theta_s + J\theta_x + \frac{2}{3}\left(\frac{J}{\rho}\right)_x \rho\theta - \frac{2}{3}\left(\kappa_0\theta_x\right)_x = \left(\frac{2}{3} - \frac{\tau_m^2}{3\tau_m\tau_e}\right)\frac{J^2}{\rho} - \frac{\rho}{\tau_m\tau_e}(\theta - \bar{\theta}), \tag{H'3}$$

$$\phi_{xx} = \rho - D. \tag{H'4}$$

Making the square τ_m^2 tend to zero with the product $\tau_m\tau_e$ kept constant in (H') yields the energy transport model

$$\rho_s + J_x = 0, \tag{E1}$$

$$\rho\theta_s + J\theta_x + \frac{2}{3}\left(\frac{J}{\rho}\right)_x \rho\theta - \frac{2}{3}\kappa_0\theta_{xx} = \frac{2}{3}\frac{J^2}{\rho} - \frac{\rho}{\tau_m\tau_e}(\theta - \bar{\theta}), \tag{E2}$$

$$\phi_{xx} = \rho - D \tag{E3}$$

with the electric current $J = \rho\phi_x - (\theta\rho)_x$. Furthermore letting both of the relaxation times τ_m and τ_e tend to zero in (H') or in (E), we have the drift-diffusion model

$$\rho_s + J_x = 0, \tag{D1}$$

$$\phi_{xx} = \rho - D \tag{D2}$$

with the electric current $J = \rho\phi_x - (\bar{\theta}\rho)_x$.

We show that all models (H'), (E) and (D) admit unique stationary solutions satisfying boundary conditions and the time global solutions for the initial boundary value problems provided that the boundary strength $|\rho_l - \rho_r| + |\phi_r|$ is sufficiently small. In addition, it is discussed that the asymptotic behaviors of solutions for the models are given by the corresponding stationary solutions. The formal computations of the relaxation time limits are also rigorously justified. Precisely we show that the time global solution for initial boundary value problem (H'), (I) and (B) converges to the solution for (E) if $\tau_m^2 \to 0$ and $\tau_m\tau_e =$ constant. Moreover the solutions for (H') and (E) converge to that for (D) if $\tau_m \to 0$ and $\tau_e \to 0$. In these limit procedures we have to handle the initial layer problem, which occurs as the initial data (I) is not necessarily in the equilibrium states for (E) and (D). Notice that we can only prescribe two initial conditions for (E) and one initial condition for (D) while three initial conditions in (I) are necessary for (H'). However, the layers are shown to decay as the time t tends to infinity and/or the relaxation times τ_m and τ_e tend to zero. In the all results above, any smallness assumptions on the initial data are necessary provided that the relaxation times are sufficiently small.

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