Hyperbolic Models for Large Supply Chains

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A supply network is defined as a set of nodes, each processing a product and passing it along to other suppliers along the arcs of a graph until reaching a final customer [5]. This talk will give an overview over recent developments, modeling the flow of products through the network by hyperbolic conservation laws [1], [2].

Models for supply networks obviously share a lot of features with traffic flow models. These shared features are

- There are conserved quantities (products and vehicles).
- Conservation laws are defined on the arcs of a graph instead of in continuous space [8], [9].
- Stochasticity, i.e. the random behavior of drivers or suppliers, is a key factor in the behavior of the system [4].
- The flux functions of these conservation laws can be derived from microscopic descriptions of the individual behavior of vehicles ('follow the leader' models [3]) or suppliers and, consequently, from underlying kinetic descriptions of the flow [10], [11].

There are, however significant differences between the two model types:

- In supply networks product can be stored in inventories, leading to arbitrarily large densities and, consequently, to measure valued solutions of the conservation laws [2], [7].
- Modeling the decision making process of individual suppliers, i.e. the influence of service policies, is a much more important issue in the modeling of supply networks [13].

After giving an overview over various conservation law models for supply networks, this talk will focus on level set approaches to modeling scheduling policies in supply chains. The starting point is a rather high dimensional kinetic model for a density of parts / particles with multidimensional attributes. After deriving hyperbolic conservation laws for the level sets of all parts with equal scheduling priority, a solution of the kinetic equations is obtained via a multi - phase closure [12]. Random behavior of suppliers, i.e. random breakdowns in the supply chain are investigated in the case of Markovian breakdowns [6].

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