A HYBRID MESHLESS/SPECTRAL-ELEMENT METHOD FOR GEOPHYSICAL FLUID DYNAMICS ON THE SPHERE

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The purpose of this paper is to present a hybrid numerical technique that couples spectral element approximation methods with meshless collocation methods for use in solving large-scale initial-boundary value partial differential equations from geophysical fluid problems on the sphere. The interest in constructing such a hybrid numerical method for large-scale problems is namely to capture the robust high-order approximation properties of nodal Lagrangian basis spectral element methods with the versatile approximation properties of meshless collocation. We show that with such a hybrid formulation, meshless collocation approximation can be utilized to refine regional approximation of global numerical solutions in fluid problems in lieu of traditional grid refinement techniques that are used in finite/spectral element methods. Furthermore, we show that this use of meshless collocation ultimately leads to faster refinement due to the fact that no mesh is needed.

Coupled with the nodal spectral element method is the Empirical Backus-Gilbert meshless collocation method developed by Blakely in [1] which was shown to be versatile in choosing the approximation space of the method while endowed with high-order accuracy in the approximation of smooth problems. However, as with other meshless collocation methods, high collocation node counts in the computational domain renders the method ill-conditioned thus making it computationally infeasible for large-scale problems on the sphere. Utilizing an innovative domain decomposition technique developed by Brezzi et al. called the Three-Field Formulation and adapted for coupling spectral-elements and meshless collocation in [Blakely (2006)], we demonstrate that the hybrid scheme inherits attractive properties for use in large-scale geophysical fluid problems on the sphere without succumbing to the ill-conditioned nature of meshless collocation methods. Numerical examples will be given using standard test problems of the shallow-water equations on the sphere from Williamson et al.

References

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