The post-adiabatic correction to the phase of gravitational wave for quasi-circular extreme mass-ratio inspirals.

Based on unpublished, still progressing works

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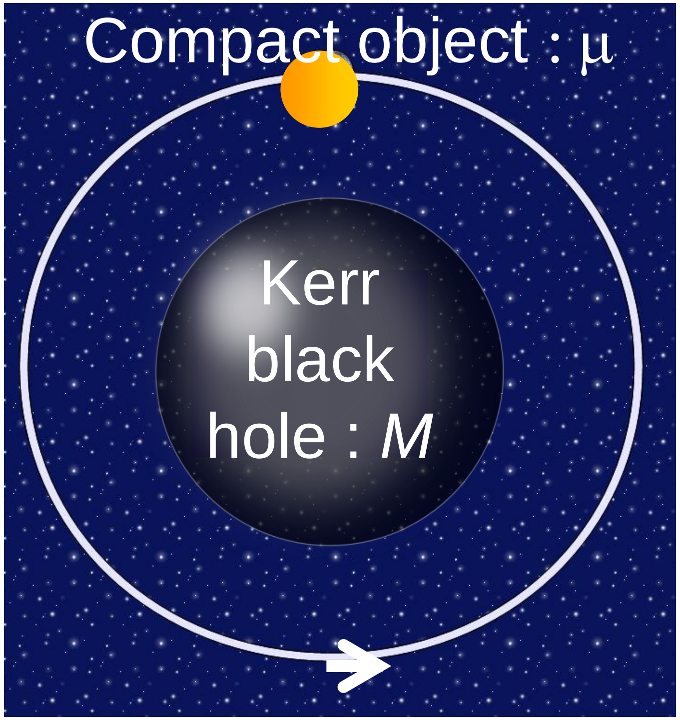
Extreme Mass Ratio Inspiral

A compact object inpirals into a more massive black hole: Promising sources of gravitational waves (GWs) for eLISA DECIGO/BBO (2020?)



The considered EMRI

- A Kerr black hole and an object

$$q := a/M \quad \nu := (M\mu)/(M+\mu)^2$$

- The object in **qusi-**circular orbit

  (The same direction of B.H. spin)

The gravitational waves from EMRI allows us to test Relativity near a black hole.

Why self-forces (SFs)?

Accurate prediction of the wave form is very welcomed

[T.Hinderer and E. Flanagan (2008)]

The total phase of GWs

$$\Phi := \frac{M^2}{\mu}\left[\Phi^{(0)}(\tilde{t}) + \frac{\mu}{M}\Phi^{(1)}(\tilde{t}) + O\left(\frac{\mu}{M}\right)^2\right]$$

$\tilde{t} := (\mu/M)t$

$\Phi^{(0)}(\tilde{t})$

- Averaged 1st order dissipative SFs

$\Phi^{(1)}(\tilde{t})$

- Oscillating 1st order dissipative SFs
- 1st order conservative SFs

[N. Warburton+ (2011) , K. Lackeos and L. Burko(2012)]

- Averaged 2nd order dissipative SFs

[E.Rosenthal (2006) S.Detweiler (2011) A.Pound (2012) , S.Gralla (2012)]

What can we say the dephasing from the 2nd order dissipative self-forces?

GWs from circularized inspirals

The phase evolution of GWs from EMRIs in the last year of inspiral is related to **the particle's energy** $E^{(P)}$

$x := (m\Omega_\phi)^{2/3}$

$m = M + \mu$

Orbital dynamics : conservative SFs

$$\Phi := 2\int_{x_{\rm ISCO}}^{x_0} dx \frac{x^{3/2}}{m} \frac{dE^{(P)}(x)/dx}{dE^{(P)}(x)/dt}$$

Energy loss via GWs : dissipative SFs

The relative order from the leading term is different

$$\dot{E}^{(P)} := \dot{E}^{(1)}(1 + \nu\dot{E}^{(2)} + O(\nu^2))$$

$$E'^{(P)} := E'^{(G)}(1 + \nu E'^{(1)} + O(\nu^2))$$

$\tilde{t} := (\mu/M)t$

Geodesics

1st SFs

2nd SFs

Incorporate into the PN theory

Borrow partial knowledge from the PN formalism

$$\Phi := 2\int_{x_{\rm ISCO}}^{x_0} dx \frac{x^{3/2}}{m} \frac{dE^{(P)}(x)/dx}{\boxed{dE^{(P)}(x)/dt}} \qquad x := (m\Omega_\phi)^{2/3}$$

We assume the rate that energy is lost through GWs is equal to the rate that the SFs remove energy from the orbit.

Balance argument for phase evolution

GW energy flux emitted to the infinity

(and to a Kerr black hole: suppressed)

$$-\left\langle \frac{dE^{(P)}}{dt} \right\rangle = \mathcal{L}_\infty + \mathcal{L}_{\rm H} \qquad \mathcal{L}_{\rm H} \leq 10^{-1}\mathcal{L}_\infty$$

$$(a = 0.998M)$$

Dissipative SFs as GWs energy flux

The effects of dissipative SFs in circular orbits can be read out from the Taylor expanded energy flux.

$$\mathcal{L} := \frac{32}{5}\nu^2 x^5 \left(\mathcal{L}^{(0)}(x) + \nu\mathcal{L}^{(1)}(x) + O(\nu^2)\right) \quad x := (m\Omega_\phi)^{2/3}$$

The flux from a particle in circular orbit around a Kerr black hole is ($\mathcal{L}^{(1)}$: only linear spin coupling terms).

[T.Tagoshi (1996)]

$$\mathcal{L}^{(0)}(q) = \sum_{k=0}^{4}\sum_{p=0}^{[k/3]} F^{(k,\,p)}(q)x^k(\log(x))^p + O(x^{9/2})$$

[L.Blanchet+ (2011)]

$$\mathcal{L}^{(1)}(q) = \sum_{k=0}^{3} G^{(k)}_{\mathrm{liniar}}(q)x^k + O(x^{7/2})$$

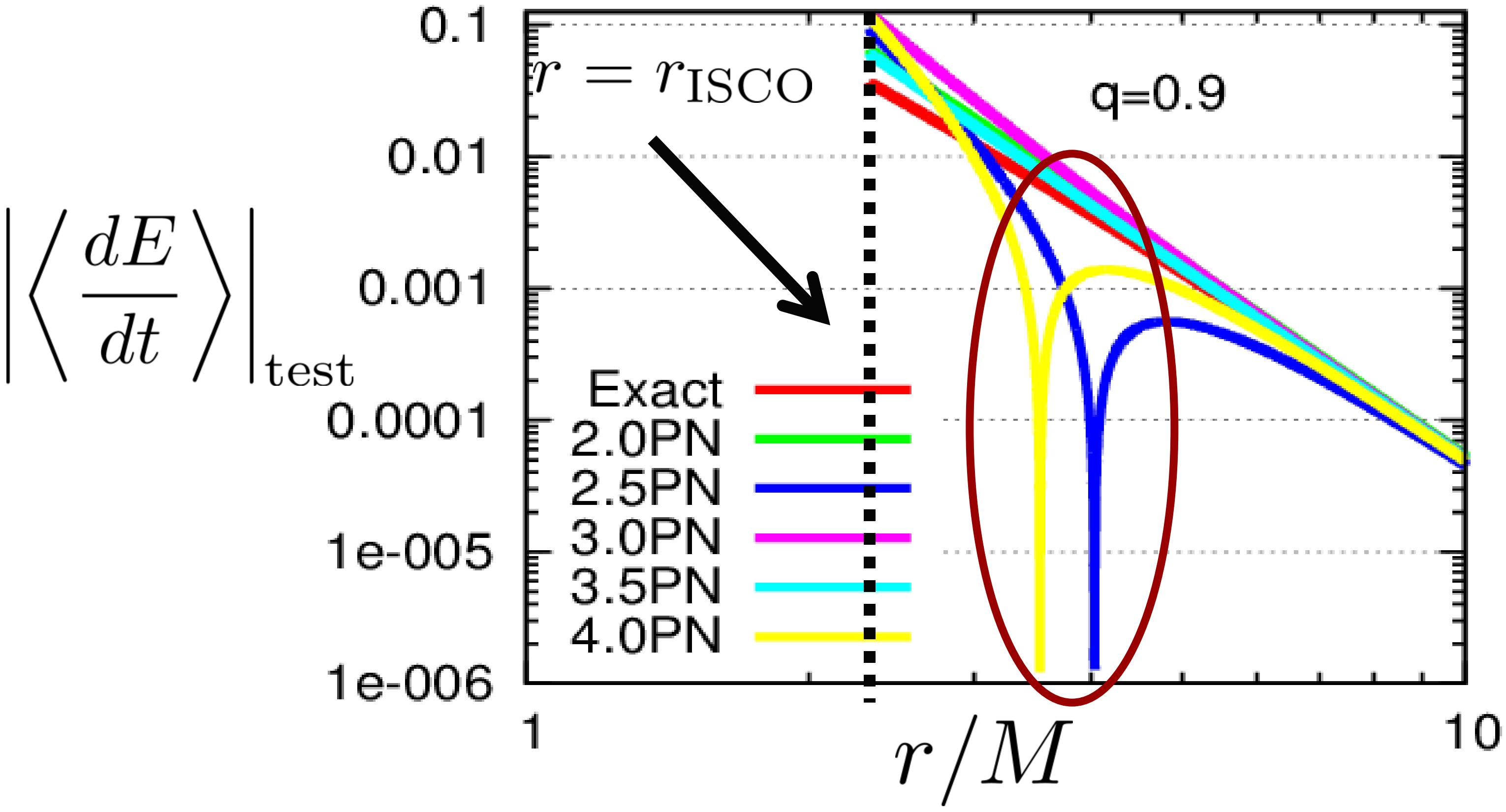
Need full dissipative 2nd order SFs

Dissipative 2nd order SFs but evaluated within PN formalism

[E.Huerta and J. Gair (2009) , N. Yunes+(2011)]

Energy flux can be negative

Taylor expanded flux with spin dependent terms becomes **negative** outside the ISCO radius in the test particle limit.



Need to cure the negative flux before calculation.

Exponential resummation. 1

The GWs energy flux should be positive definite

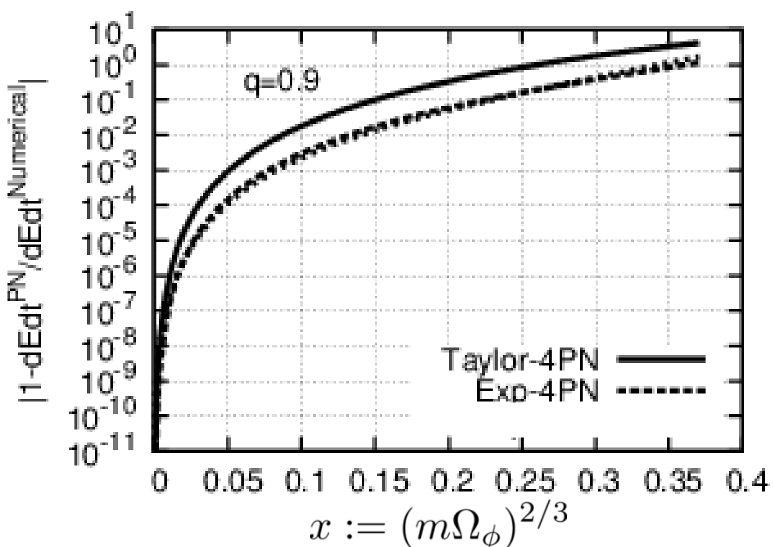
Exponential resummation in the test particle limit

$$\frac{32}{5}\nu^2 x^5 \mathcal{L}^{(0)}(x) \Rightarrow \mathcal{F}^{(0)}(x) = \frac{32}{5}\nu^2 x^5 \exp\left[\log(\mathcal{L}^{(0)}(x))\right]$$

Positive definite

$$\mathcal{L}^{(0)}(q) = \sum_{k=0}^{4}\sum_{p=0}^{[k/3]} F^{(k,\ p)}(q) x^k (\log(x))^p + O(x^{9/2}) \qquad x := (m\Omega_\phi)^{2/3}$$

Exponential resummation improves the accuracy of the analytic energy flux, at the same time.



Exponential resummation. 2

We also introduce a exponential resummation with finite mass correction.

Exponential resummation

$$\mathcal{F}^{(1)}(x) = \frac{32}{5}\exp\left[\log\left(\mathcal{L}^{(0)}(x)\right) + \log\left(1 + \nu\frac{\mathcal{L}^{(1)}(x)}{\mathcal{L}^{(0)}(x)} + O(\nu^2)\right)\right]$$

Positive definite

$(\nu \ll 1)$

$$\Rightarrow \mathcal{F}^{(1)}(x) = \frac{32}{5}\nu^2 x^5 \mathcal{L}^{(0)}(x)\exp\left(\nu\frac{\mathcal{L}^{(1)}(x)}{\mathcal{L}^{(0)}(x)}\right)$$

Moreover, test particle sector can be replaced with the exact Teukolsky flux: **Hybrid flux**

$$\mathcal{F}^{(1)}(x) = \left\langle\frac{dE}{dt}\right\rangle_{\rm Teukolsky}\exp\left(\nu\frac{\mathcal{L}^{(1)}(x)}{\mathcal{L}^{(0)}(x)}\right)$$

How to estimate “residual” correction?

$$\mathcal{F} := \frac{32}{5}\nu^2 x^5 \left(\mathcal{L}^{(0)}(x) + \nu\mathcal{L}^{(1)}(x) + O(\nu^2)\right) \qquad x := (m\Omega_\phi)^{2/3}$$

$$\mathcal{L}^{(1)}(q) = \sum_{k=0}^{3} G^{(k)}_{\rm liniar}(q) x^k + O(x^{7/2})$$

Residuals

Exponential resummation (Hybrid flux)

Full 2nd SFs

We try to estimate the phase correction from “residuals” part of the flux via following extrapolation.

Estimator for the residuals

$$\Delta\Phi_{2nd} := \Delta\Phi^{(1)}_{\rm PN} \times \left(\frac{\Phi^G - \Delta\Phi^G_{\rm PN}}{\Delta\Phi^G_{\rm PN}}\right)$$

$$\Phi^G := 2\int_{x_{\rm ISCO}}^{x_0} dx \frac{x^{3/2}}{m}\frac{dE^G(x)/dx}{\mathcal{F}_{\rm Teukolsky}}$$

$$\Delta\Phi^G := 2\int_{x_{\rm ISCO}}^{x_0} dx \frac{x^{3/2}}{m}\frac{dE^G(x)/dx}{\mathcal{F}^{(0)}_{\rm exp}}$$

Is it acceptable ??

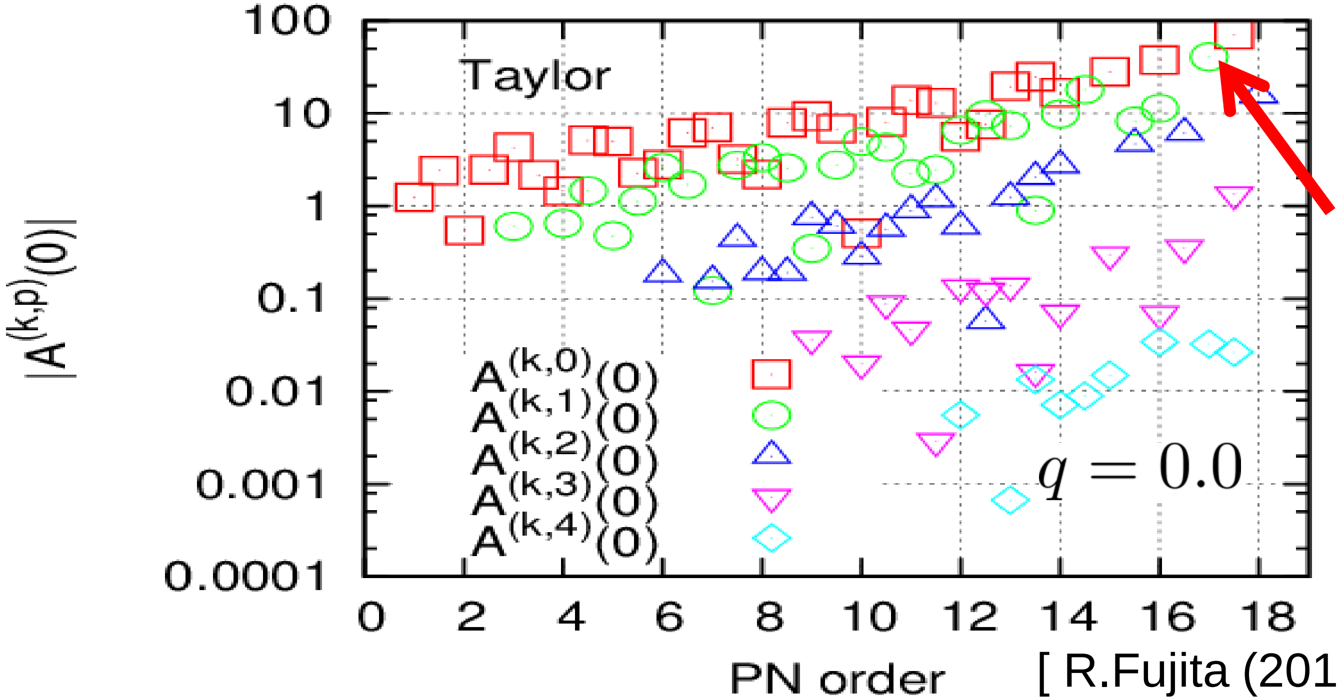
$$\Delta\Phi^{(1)}_{\rm PN} := 2\int_{x_{\rm ISCO}}^{x_0} dx \frac{x^{3/2}}{m}\frac{dE^G(x)/dx}{\mathcal{F}^{(1)}_{\rm exp}} - \Phi_G$$

Scaling law of the Coefficients in the flux

Normalize with the orbital frequency at the light ring since the source term of Teukolsky equation diverges there.

[C.Culter+ (1993) T.Damour+ (1999)]

$$\mathcal{L}^{(0)}(x) = \sum_{k=0}^{18} \sum_{p=0}^{[k/3]} A^{(k,\ p)}(q=0) \left(\frac{x}{x_{\rm pole}}\right)^k (\log(x))^p \quad x_{\rm pole} := (m\Omega_\phi^{\rm pole})^{2/3}$$



$$\frac{\overline{|A^{(k,0)}(k)|}}{\propto \exp(0.2k)}$$

[R.Fujita (2012) H.Nakano+ (2012)]

The coefficients scales with respect to the PN order

Spin and finite mass effect

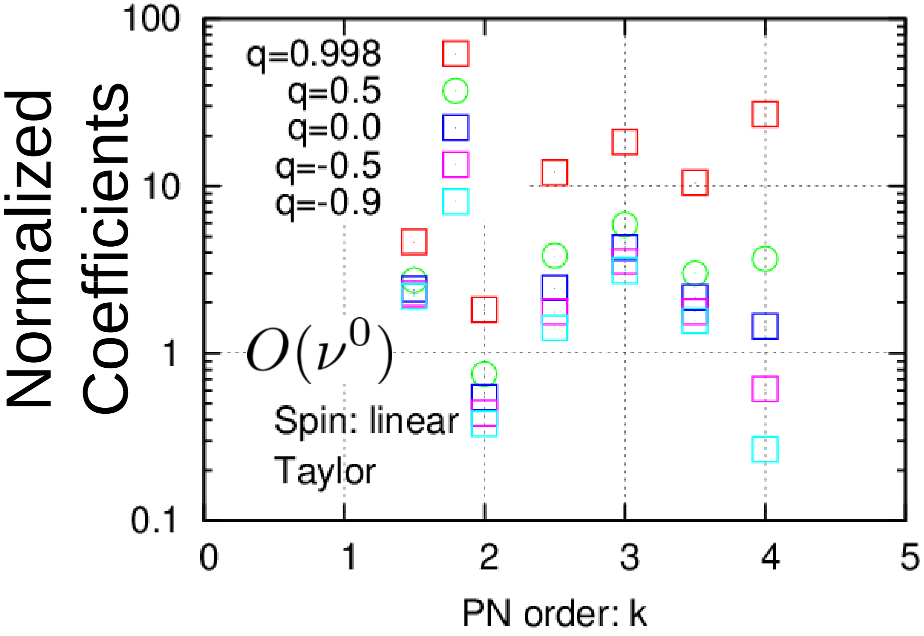
Spin and finite mass dependence in the coefficients may not ruin the scaling behavior. (**Incomplete**, however.)

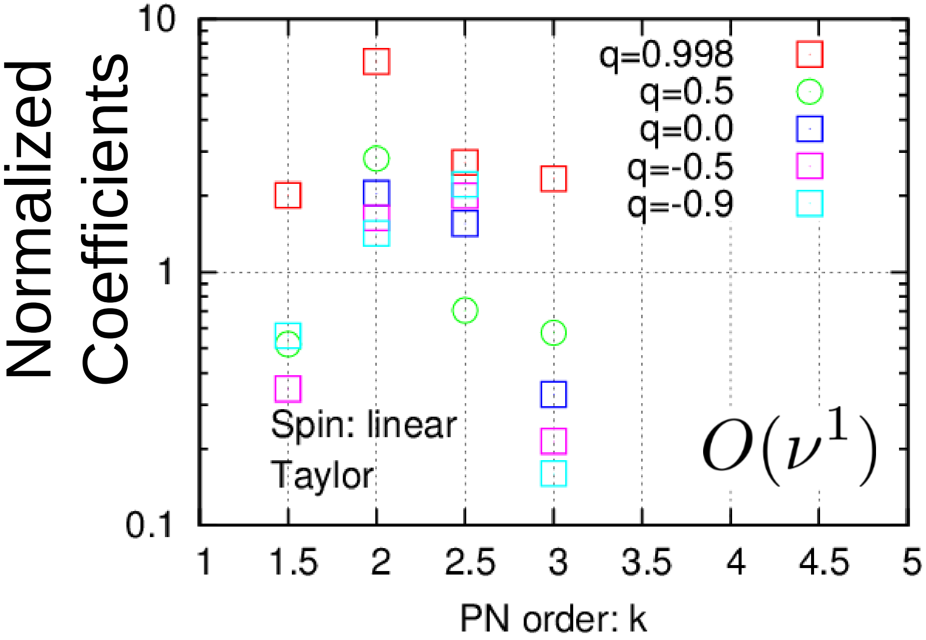
$$\mathcal{L}^{(0)}(q) = \sum_{k=0}^{4}\sum_{p=0}^{[k/3]} A^{(k,\ p)}(q)\left(\frac{x}{x_{\rm pole}}\right)^k (\log(x))^p$$

[T.Tagoshi+ (1996)]

$$\mathcal{L}^{(1)}(q) = \sum_{k=0}^{3} B^{(k)}(q)\left(\frac{x}{x_{\rm pole}}\right)^k$$

[L.Blanchet+ (2011)]





The dephasing from higher PN terms in the flux may estimated via extrapolation the one from lower PN terms.

Results

$$\Delta\Phi_{\mathrm{PN}}^{(1)} \qquad \Delta\Phi_{2nd}$$

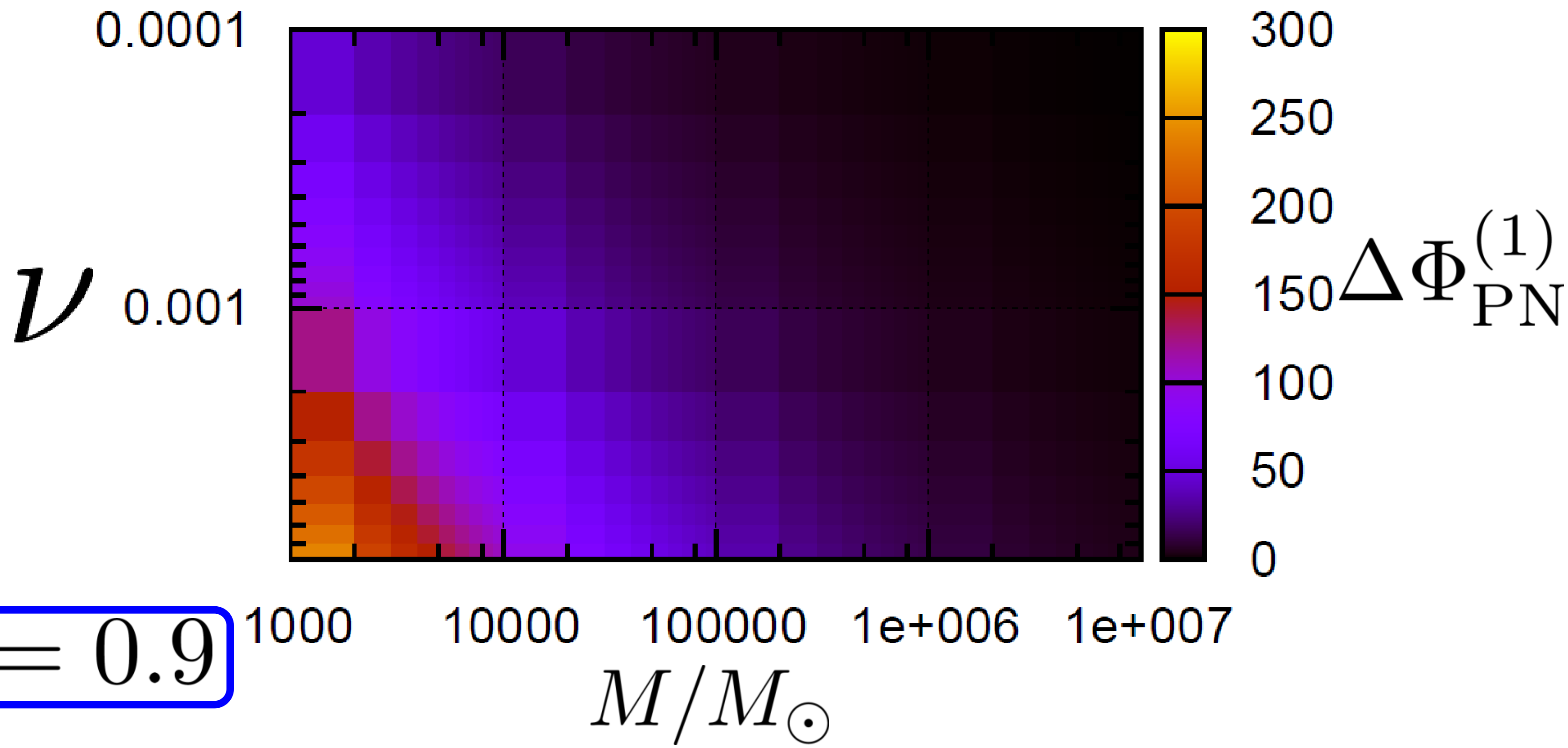
The expected phase dephasing from the dissipative part of the second order self-forces for the last year of inspiral.

(Kerr, circular orbits.)

Expected 3.0PN phase corrections

$$\Delta\Phi^{(1)}_{\rm PN} := 2\int_{x_{\rm ISCO}}^{x_0} dx \frac{x^{3/2}}{m}\frac{dE^G(x)/dx}{\mathcal{F}^{(1)}_{\rm exp}} - \Phi_G$$

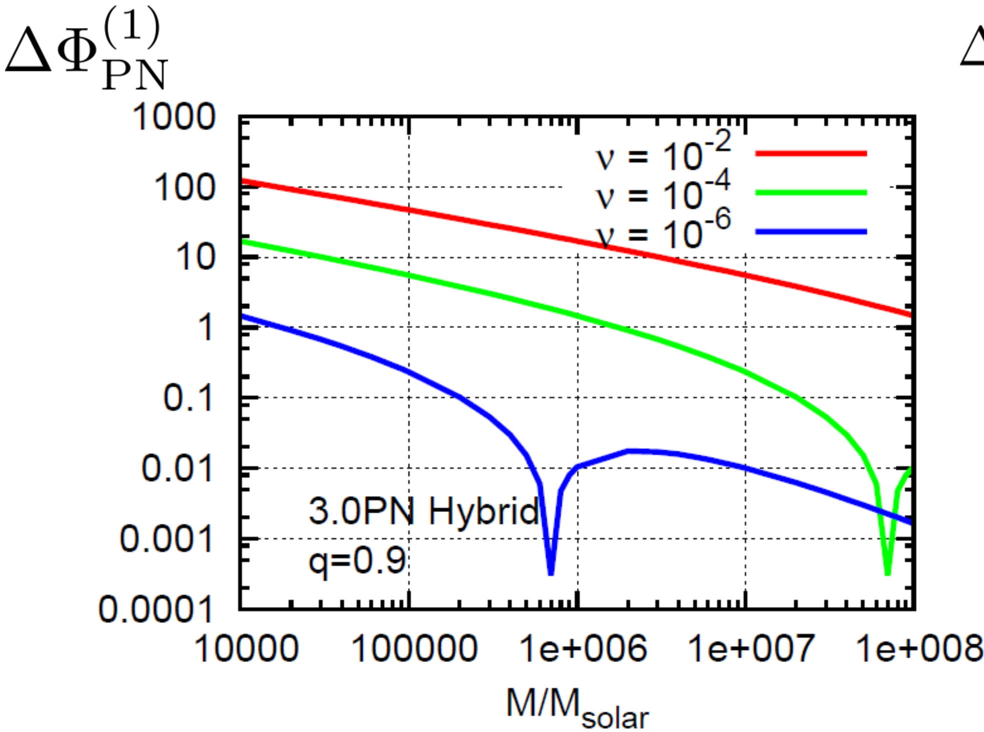
3.0PN

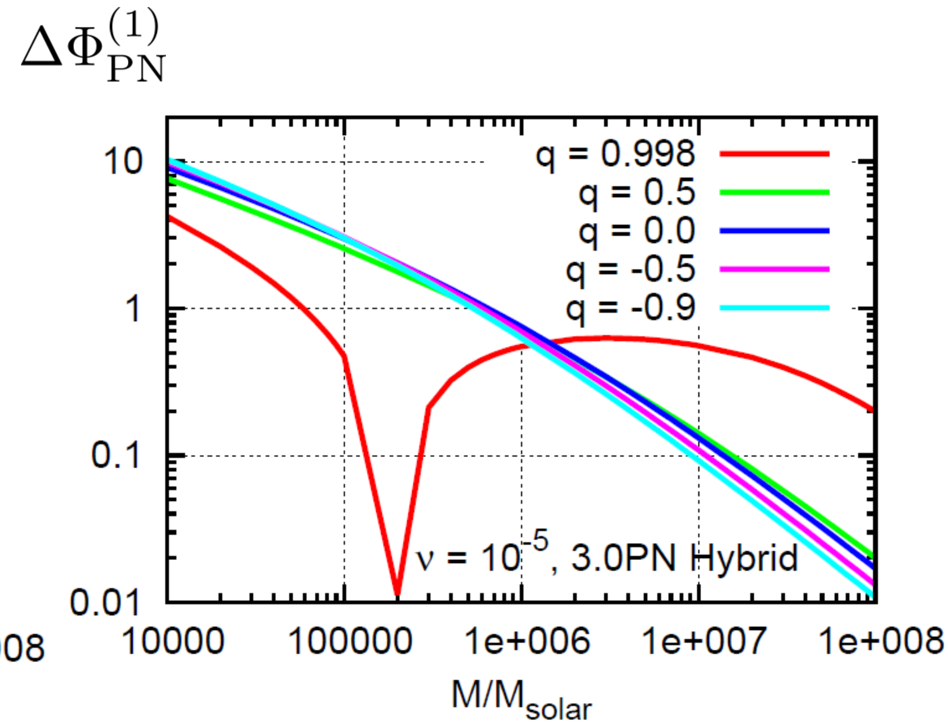


$q = 0.9$

The dephasing 2nd dissipative SFs may not be neglected for GWs from IMRIs.

Spin and mass ratio dependence

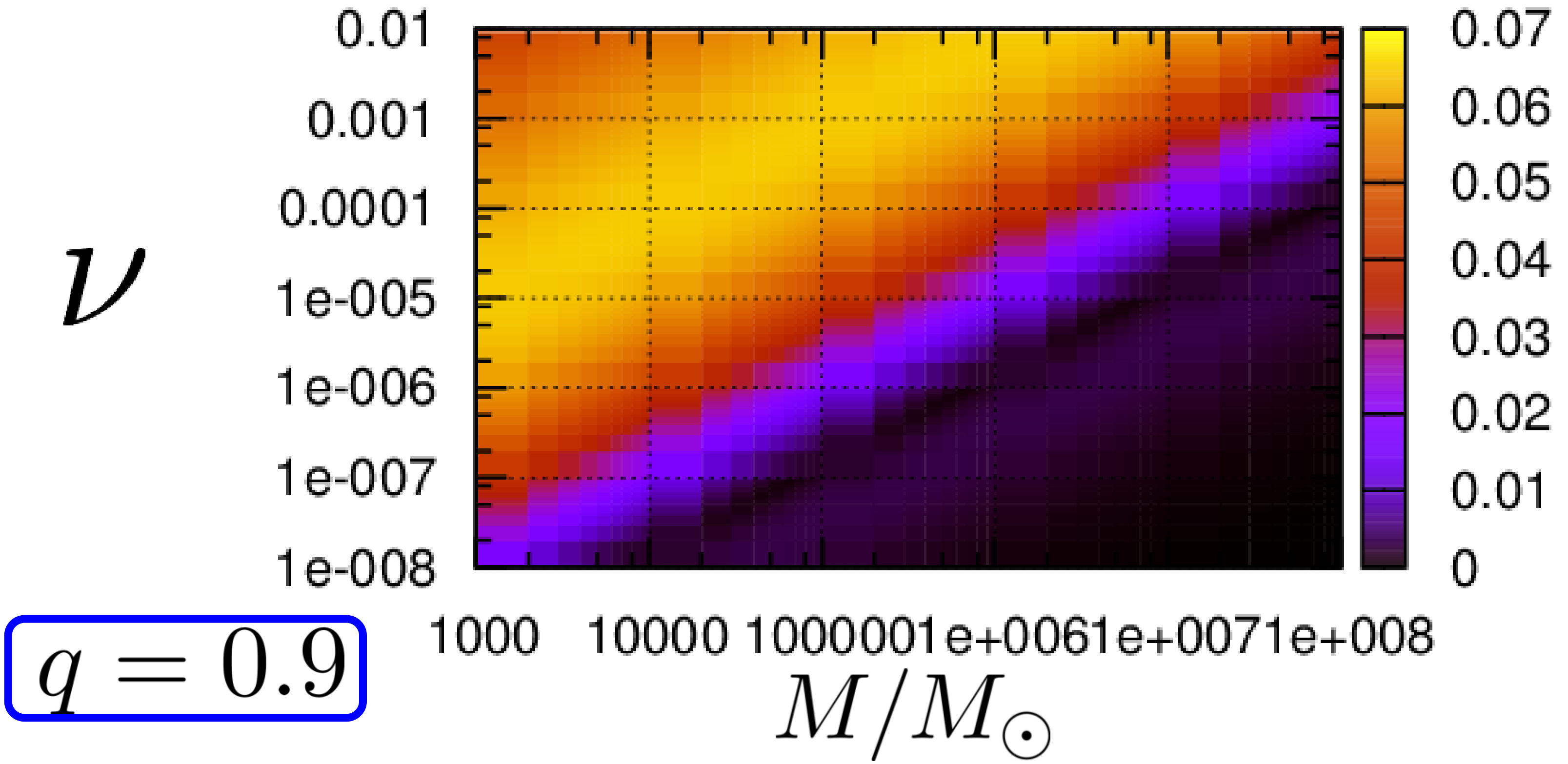




- The 3.0PN phase correction is important when the mass ratio is small.
- Spin dependence is week except the spin parameter is very close to extreme limit.

The expected dephasing from "residual" 2nd order SFs

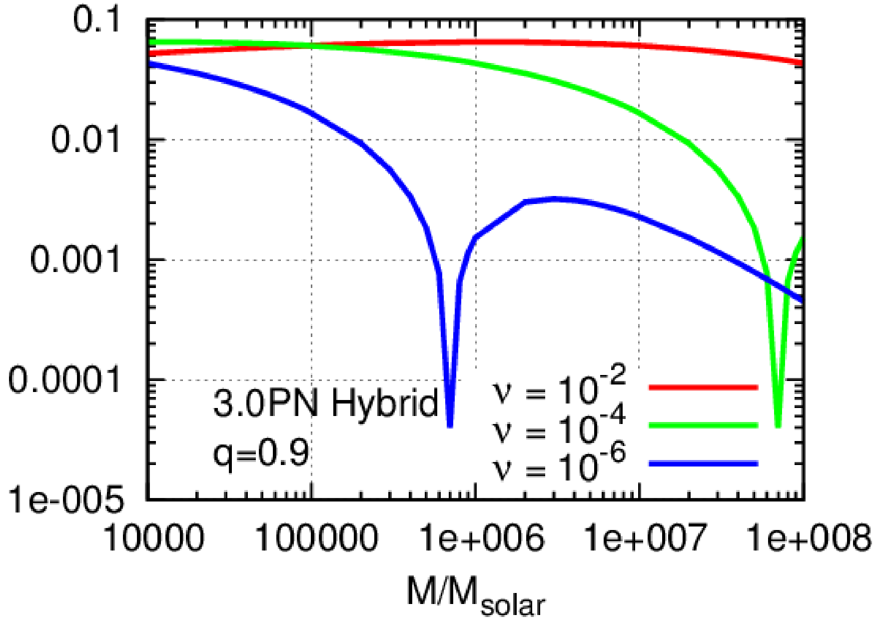
$$\Delta\Phi_{2nd} := \Delta\Phi_{\rm PN}^{(1)} \times \left(\frac{\Phi^G - \Delta\Phi_{\rm PN}^G}{\Delta\Phi_{\rm PN}^G} \right)$$



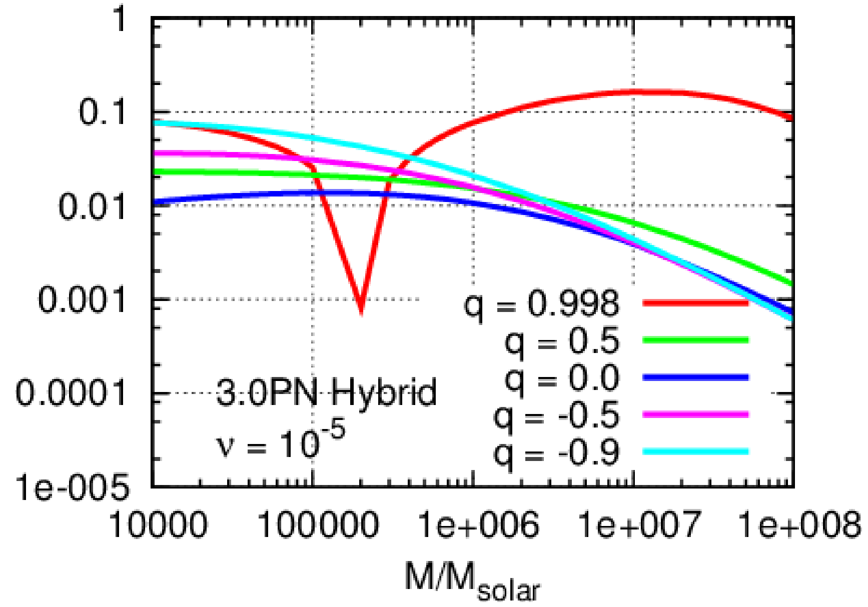
The residual dephsing from dissipative 2nd order SFs might be well suppressed among many IMRIs and EMRIs.

Spin and mass ratio dependence of “residual” dephasing

$\Delta\Phi_{2nd}$



$\Delta\Phi_{2nd}$



The suppression may hold irrespective of the black hole spin and the mass ratio of the binary.

Summary of the talk

In a circular Kerr orbit, we estimate the dephasing due to the dissipative part of the 2nd order self-forces.

- This dephasing is important for IMRIs, but they might be well captured by 3.0PN energy flux with exponential resummation.
- This dephasing coming from full 2nd order calculation may be suppressed among most IMRIs and EMRIs.

Further questions

How about a eccentric orbit?

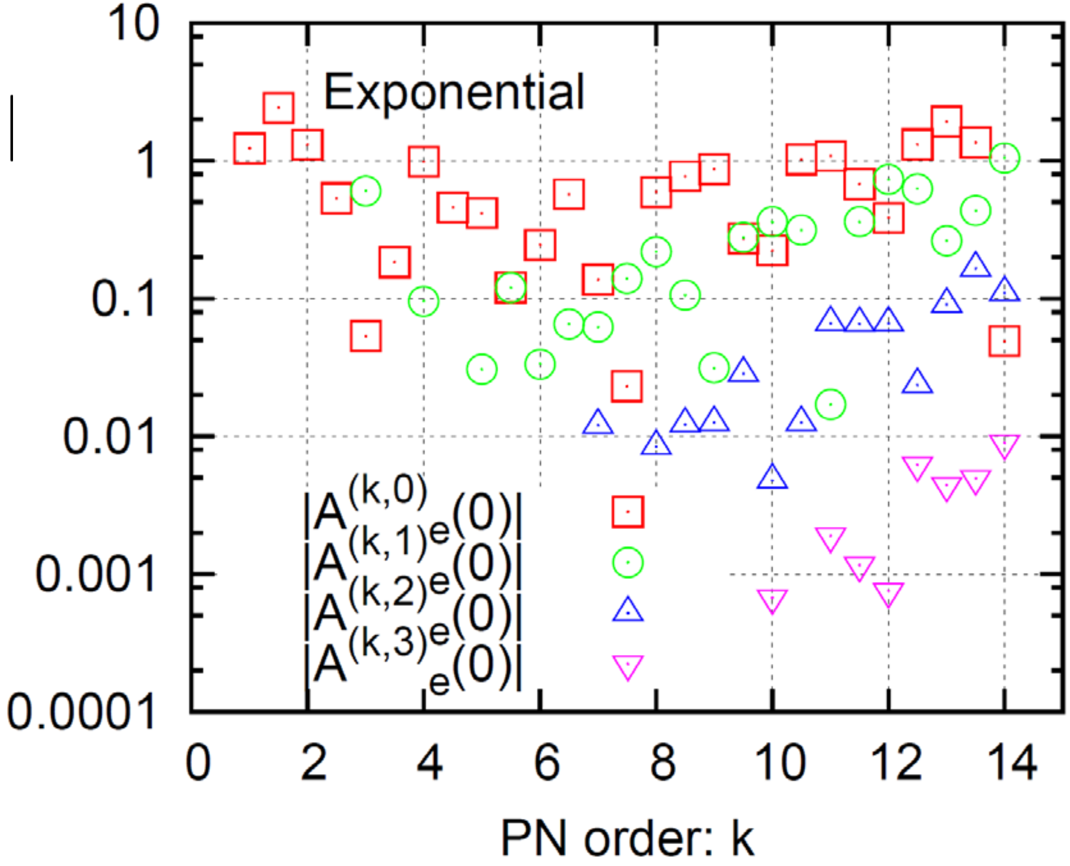
The end of planned talk

Thank you.

補遺

Scaling law of the Coefficients in the flux : Exponential resummation

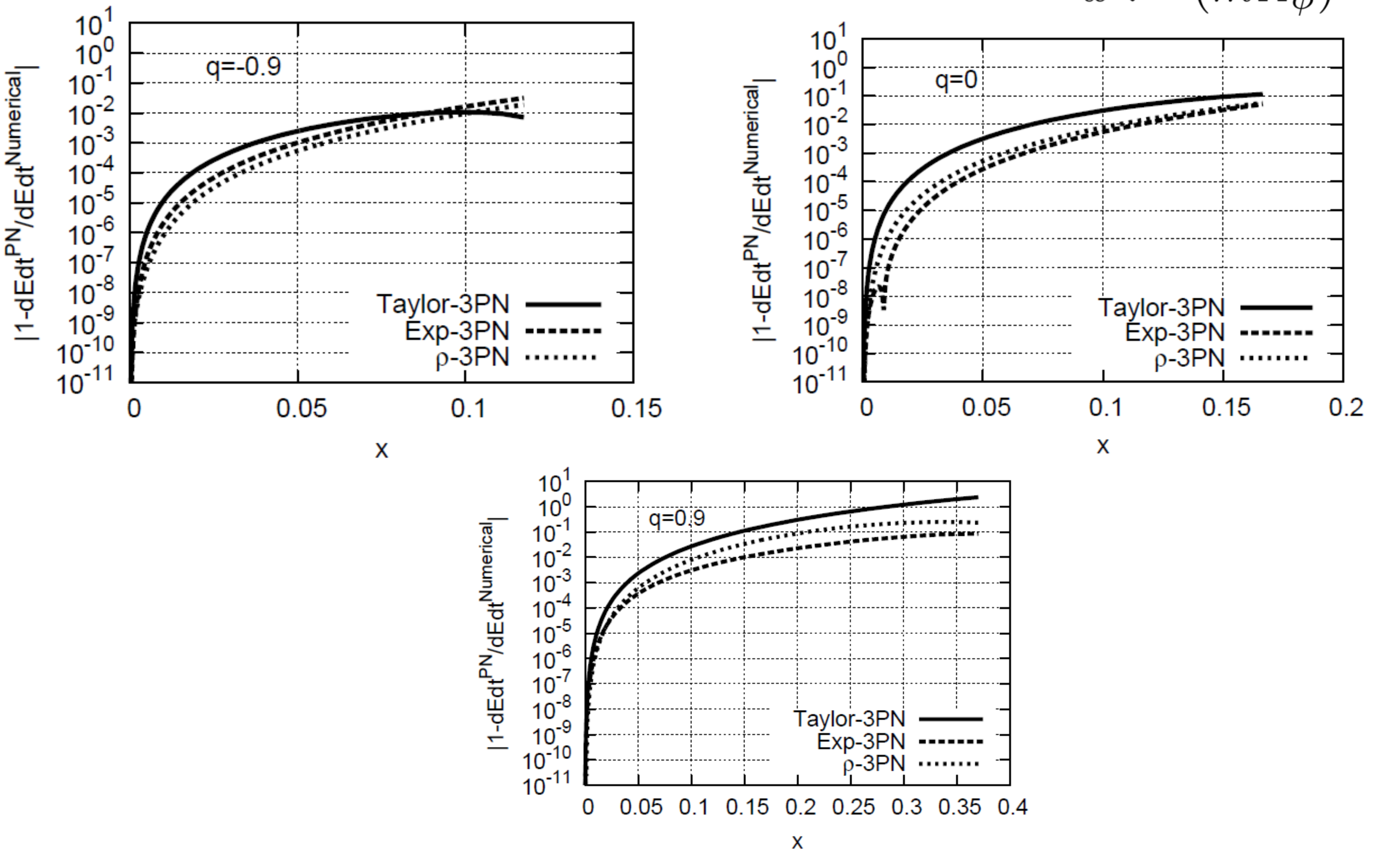
$|A_e^{(k,p)}(0)|$



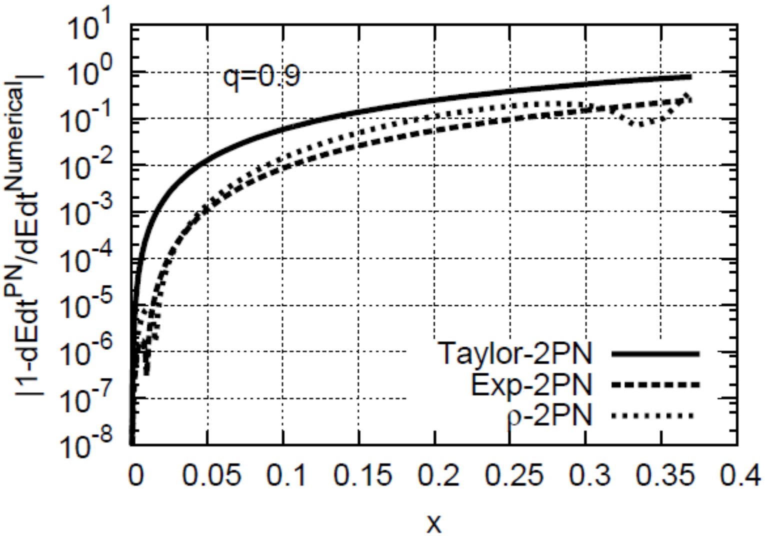
Still appears a scaling behavior at high PN order.

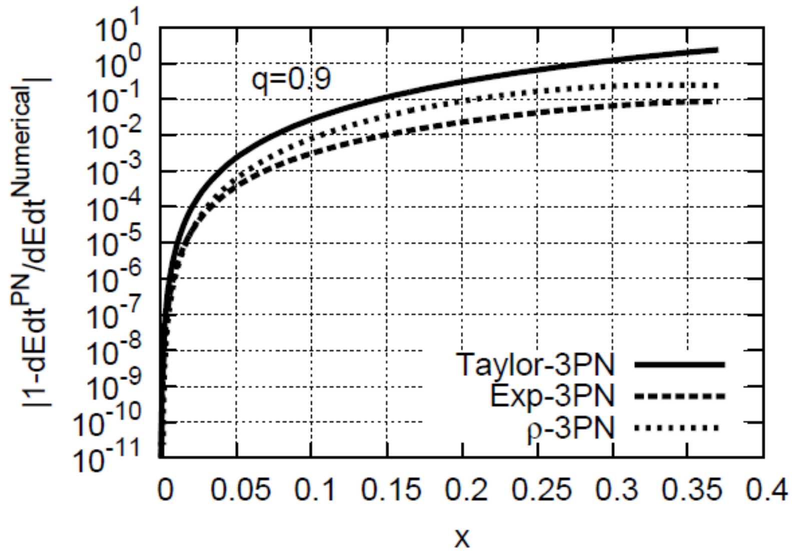
Efficiency of exponential resummation : fixed PN order

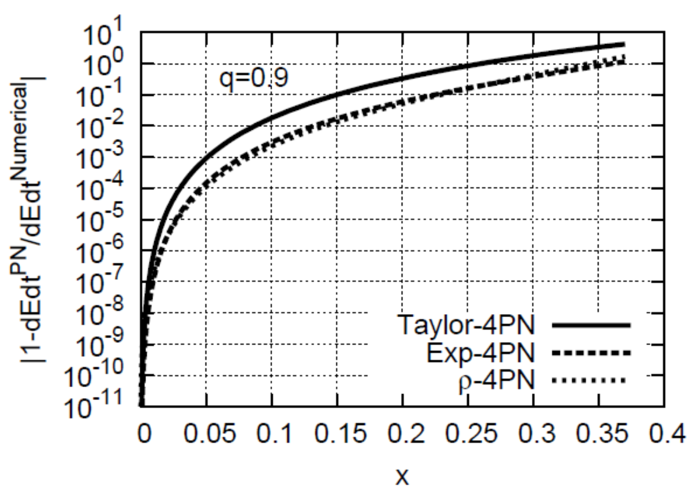
$$x := (m\Omega_\phi)^{2/3}$$



Efficiency of exponential resummation : fixed spin parameter



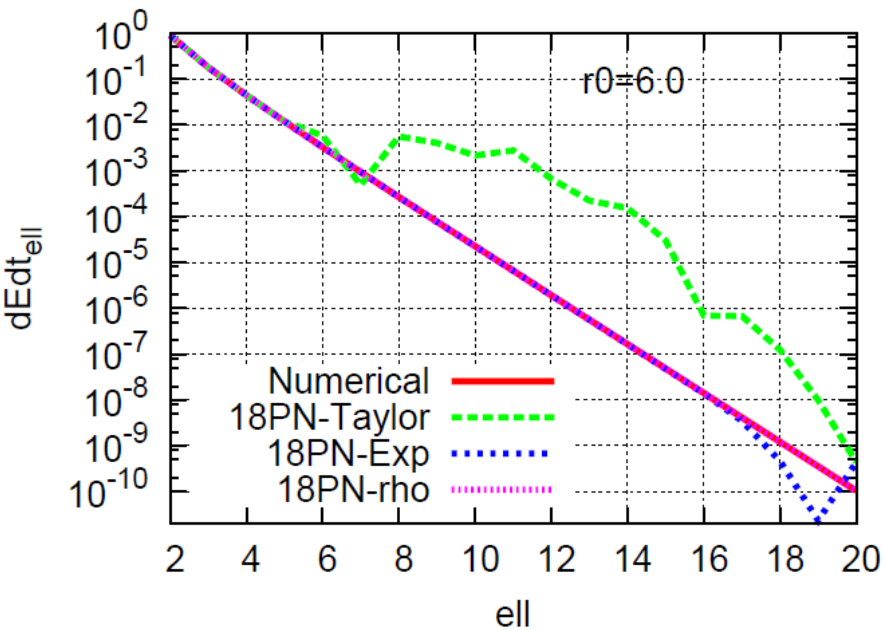


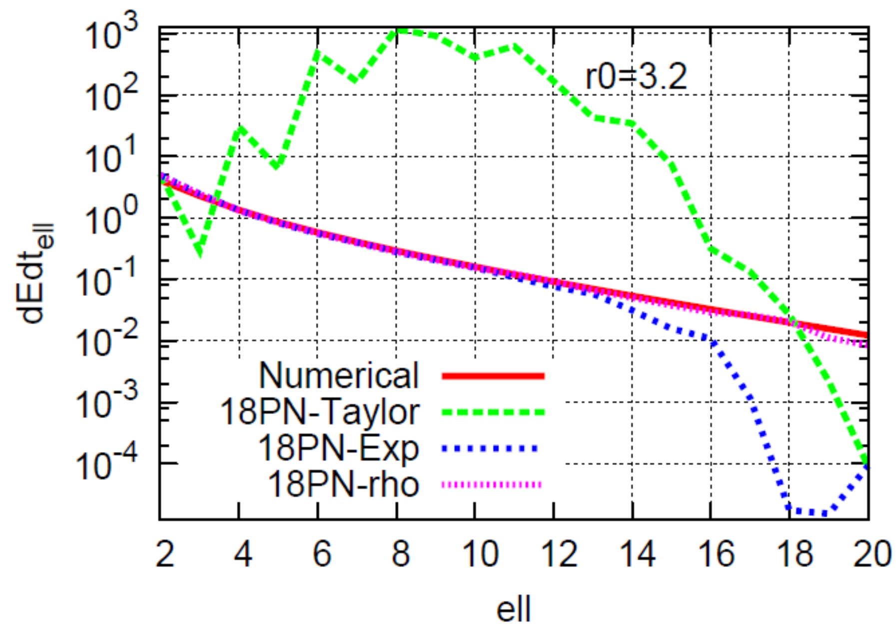


$$x := (m\Omega_\phi)^{2/3}$$

Why negative ??
(Schwarzschild case)

$$\left(\frac{dE}{dt}\right) = \frac{32}{5}\nu^2 (M\Omega_\phi)^5 \sum_\ell \left(\frac{dE}{dt}\right)_\ell$$





Inside the ISCO radius of Shwarzschild black hole, the energy flux decomposed by partial waves behaves badly if $\ell \gg 1$

Incorporate into the PN theory

Borrow partial knowledge from the PN formalism as usual

$$\Phi := 2\int_{x_{\rm ISCO}}^{x_0} dx \frac{x^{3/2}}{m}\frac{dE^{(P)}(x)/dx}{dE^{(P)}(x)/dt} \Rightarrow \Phi := 2\int_{x_{\rm ISCO}}^{x_0} dx \frac{x^{3/2}}{m}\frac{d\mathcal{E}^{(T)}(x)/dx}{d\mathcal{E}^{(T)}(x)/dt}$$

$\mathcal{E}^{(t)}$ **: total energy of the system (different from** $E^{(P)}$**)**

We assume the rate that energy is lost through GWs is equal to the rate that the SFs remove energy from the orbit.

Balance argument for phase evolution

$$-\left\langle \frac{dE^{(P)}}{dt} \right\rangle = \mathcal{L}_\infty + \mathcal{L}_{\rm H}$$

GW energy flux emitted to the infinity

(and to a Kerr black hole: suppressed) $\mathcal{L}_{\rm H} \le 10^{-1}\mathcal{L}_\infty$