# Creating horizons for light using metamaterials


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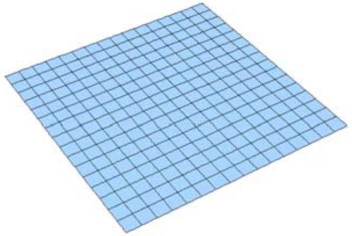
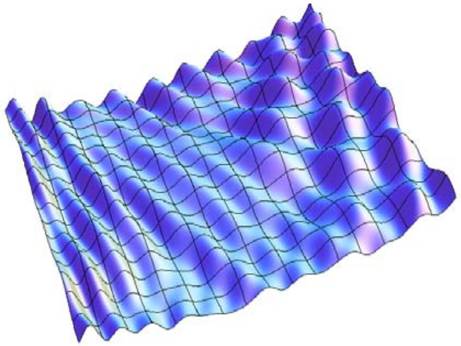
# Particles and Gravitation



In quantum field theory, the notion of a particle is founded on the Poincaré symmetry of flat space-time.

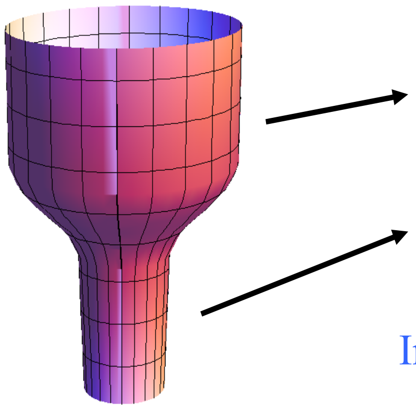
Particles: irreducible representations of the Poincaré group.



In curved space-time, no symmetry (in general).

“Particles” have no meaning in curved space-time.

Define particles in asymptotically flat regions.
Solve field equation for modes.



$$A = \sum_i \left( f_i \hat{a}_i + f_i^{*} \hat{a}_i^{\dagger} \right) \qquad \hat{a}|0\rangle = 0$$

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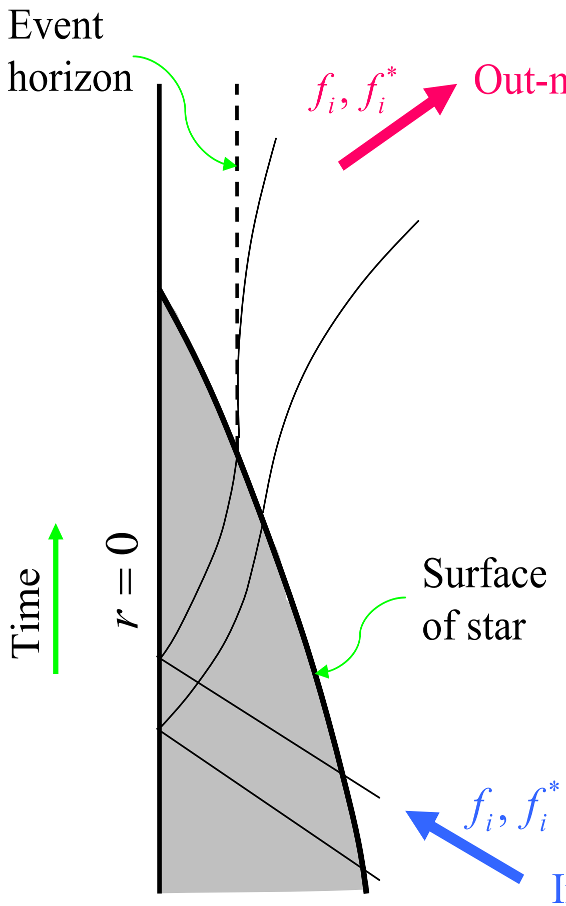
$$|0\rangle \neq |0\rangle \;!$$

Interpretation: Gravitational fields create particles.

# Collapsing Star




Out-modes (future null infinity)

$$f_i = \sum_j \left( \alpha_{ij} f_j + \beta_{ij} f_j^* \right)$$

$$\hat{a}_i = \sum_j \left( \alpha_{ij}^* \hat{a}_j - \beta_{ij}^* \hat{a}_j^\dagger \right)$$

$$\langle 0 | \hat{N}_\omega | 0 \rangle = \sum_{\omega'} \left| \beta_{\omega\omega'} \right|^2 = \left( e^{2\pi\omega/\kappa} - 1 \right)^{-1}$$

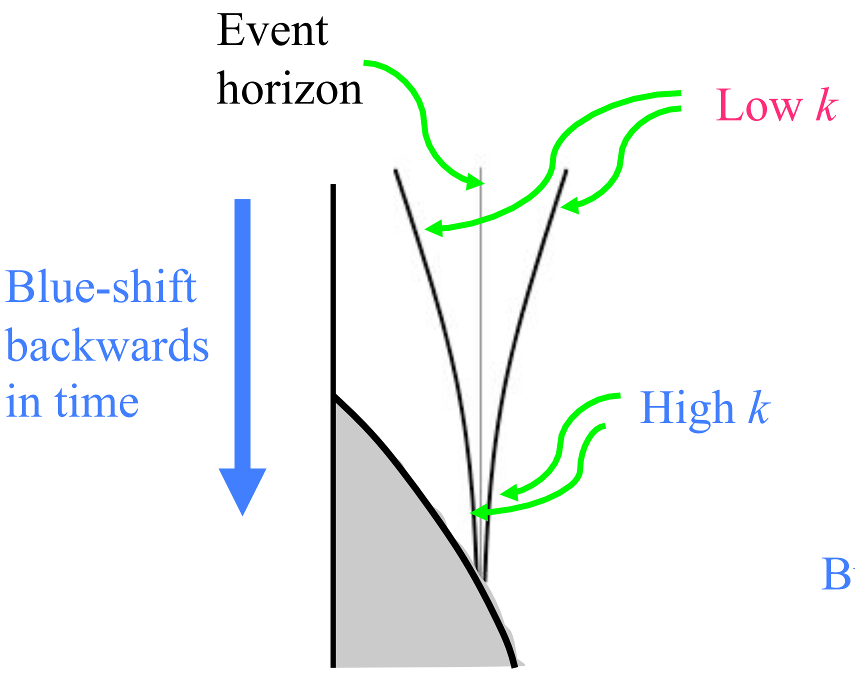
$\kappa$ – surface gravity of the black hole

Planck spectrum with temperature $k_B T = \dfrac{\hbar\kappa}{2\pi}$

In-modes (past null infinity)

*Horizons create particles.*

# Was Hawking right?



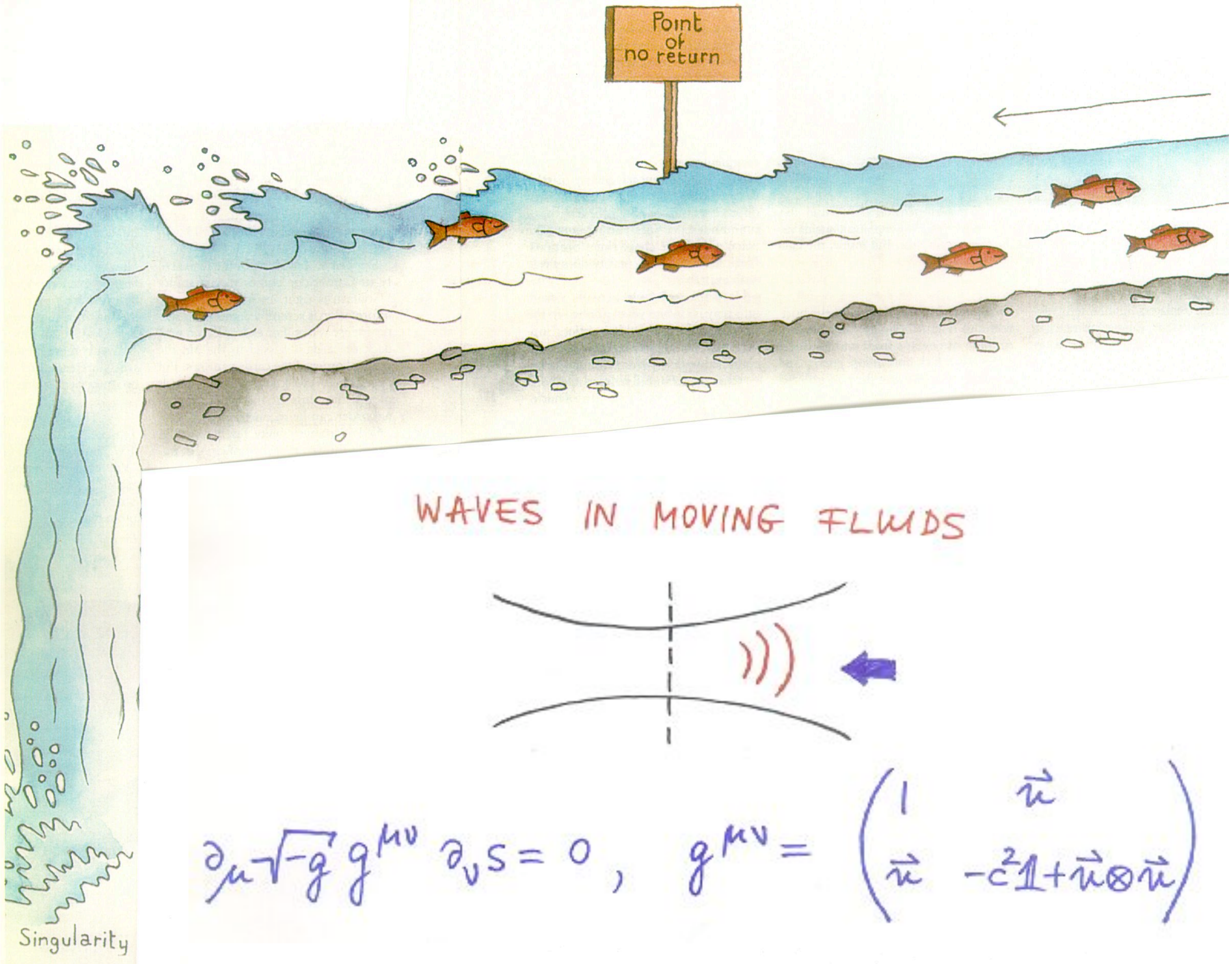

Photon wavelengths that reach a distant observer are red-shifted by a factor $\propto e^{t/4M}$

Arbitrarily large frequencies in the past
→ Arbitrarily large masses
→ Arbitrarily large gravitational effects

But we ignored back-reaction on the black hole!

The Hawking effect would appear to depend on energies beyond the Planck energy $\sqrt{\hbar c^5 / G} \approx 10^{19}\,\text{GeV}$
But we don't know how to calculate at such energies: we would need quantum gravity.

"Trans-Planckian" problem




WAVES IN MOVING FLUIDS

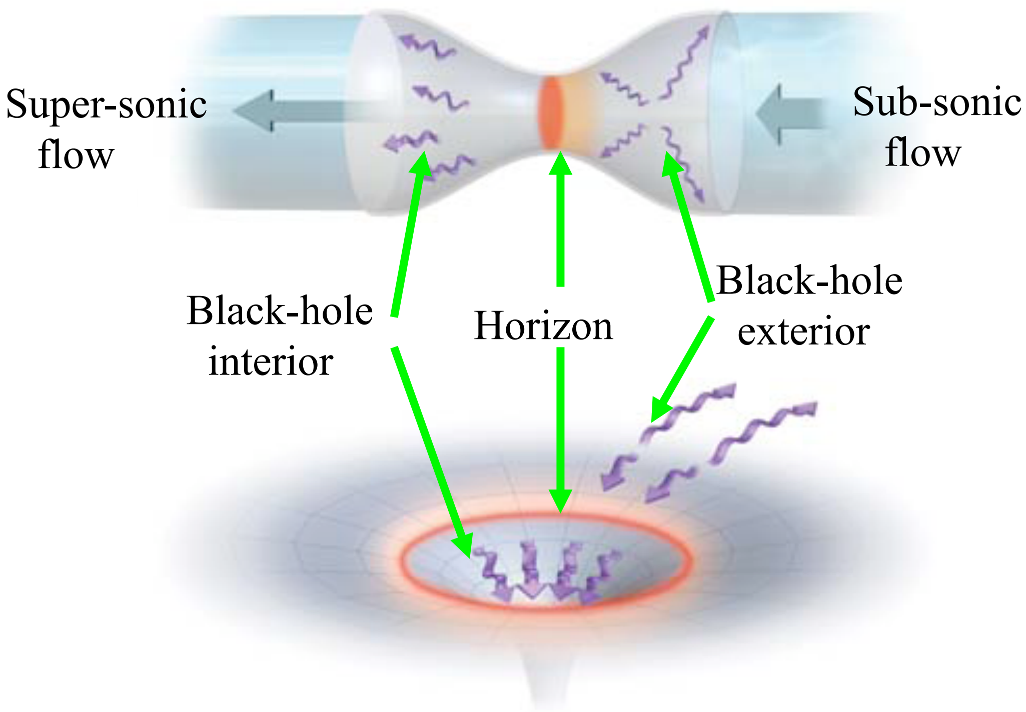
$$\partial_\mu \sqrt{-g}\, g^{\mu\nu}\, \partial_\nu S = 0 \,, \quad g^{\mu\nu} = \begin{pmatrix} 1 & \vec{u} \\ \vec{u} & -c^2 \mathbb{1} + \vec{u}\otimes\vec{u} \end{pmatrix}$$

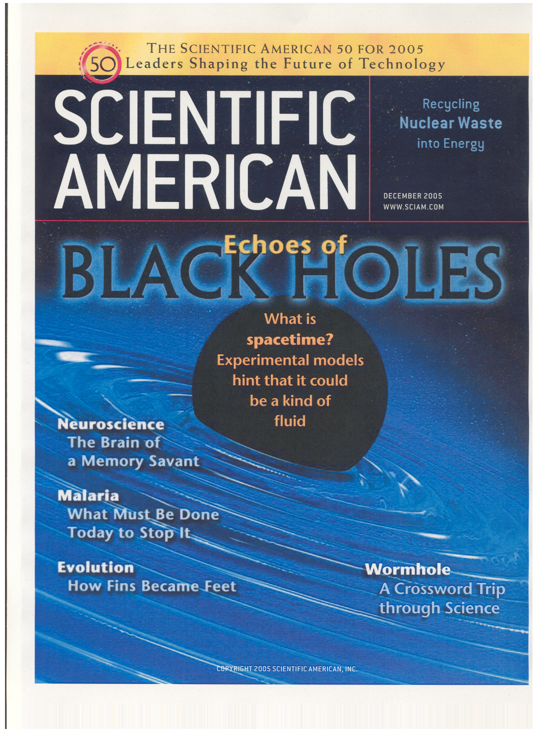
# Is space-time a fluid?

Unruh (1981): Sound propagation in a moving fluid is equivalent to a scalar field in the curved space-time given by

$$g_{\mu\nu} = \rho \begin{pmatrix} c^2 - v^2 & v^i \\ v^j & -\delta_{ij} \end{pmatrix}$$

Sonic black hole:




Standard theory of the Hawking effect predicts a thermal spectrum of phonons with temperature

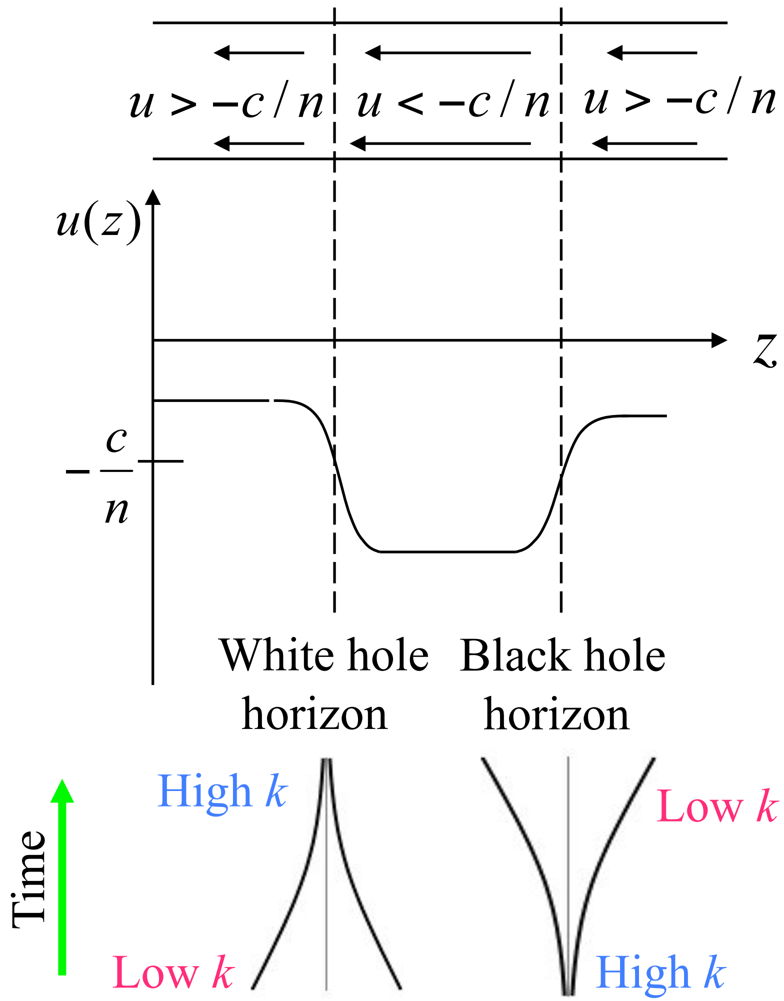
$$k_B T = \frac{\hbar\kappa}{2\pi}$$

$\kappa$ – slope of velocity ("surface gravity") at horizon

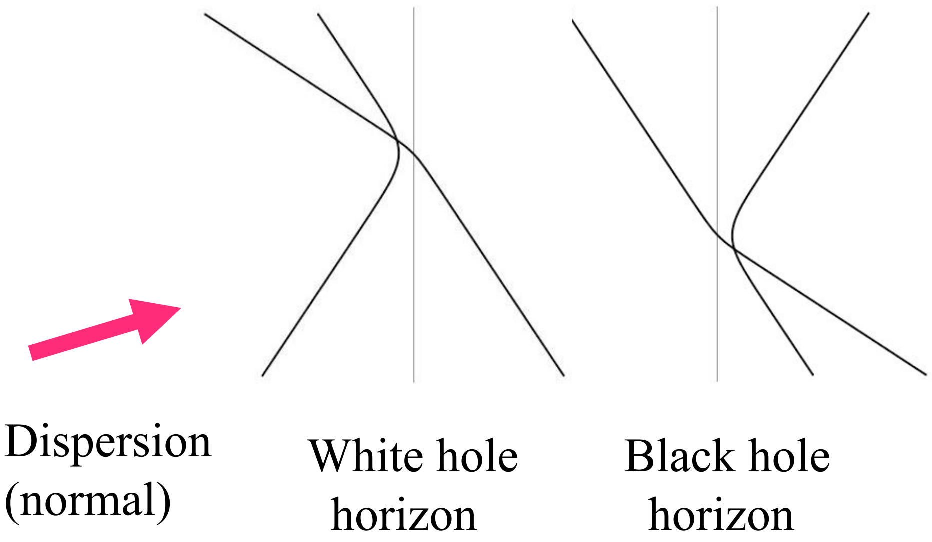
Estimate for BECs: $T \sim 10$nK.

What about "trans-Planckian" issue? This is known physics.

# White holes and black holes




Regions where speed $u$ of medium is $-c/n$ are horizons for right-moving light – analogues of black holes and white holes. White hole is time-reverse of black hole.
At horizon light is bunched up and stopped – Wavelength decreases, wave vector $k$ increases.
In real materials, this blue-shifting is limited by dispersion.



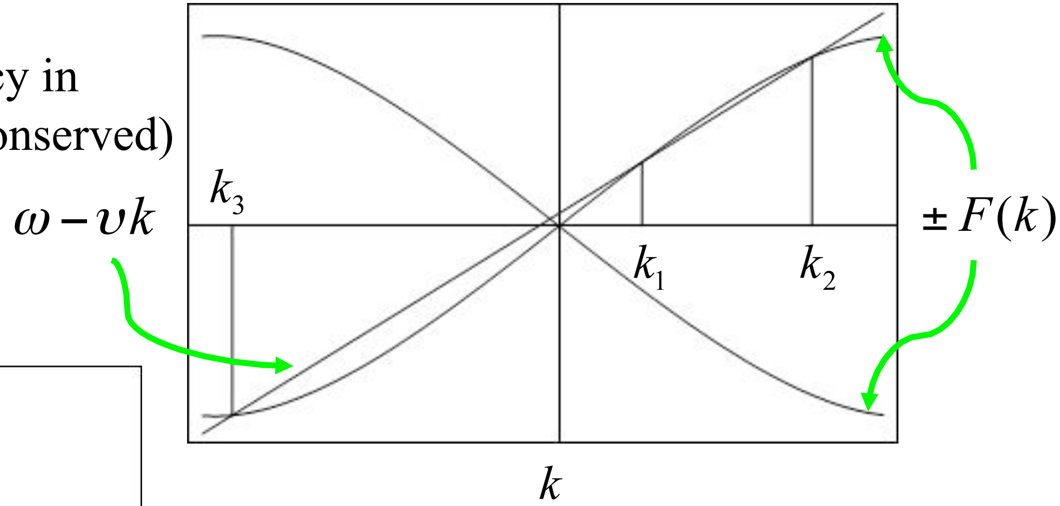
# Hawking effect with dispersion

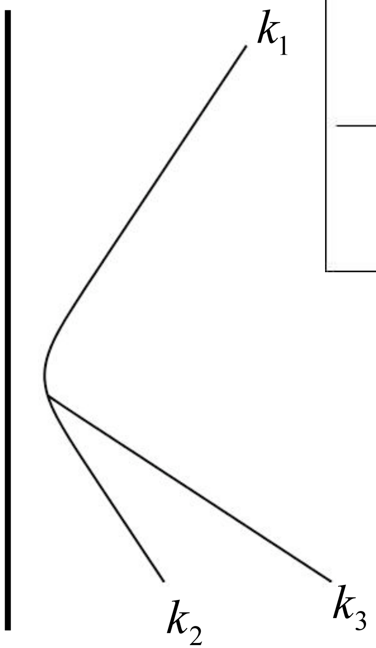
Dispersion relation:

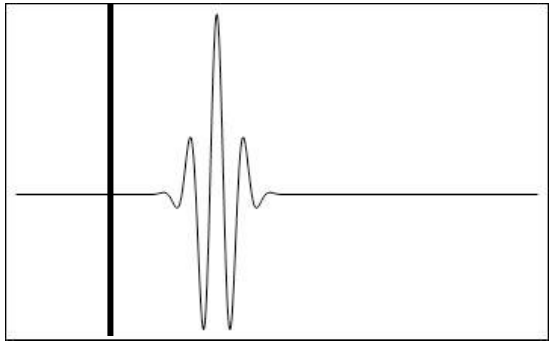
$$(\underbrace{\omega - \upsilon k}_{})^2 = F^2(k)$$

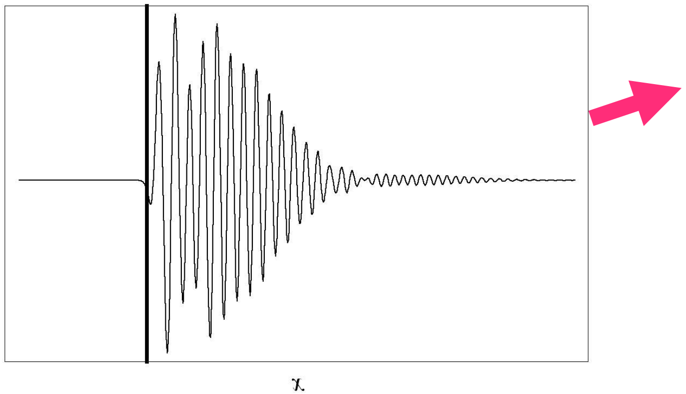
Frequency in co-moving frame

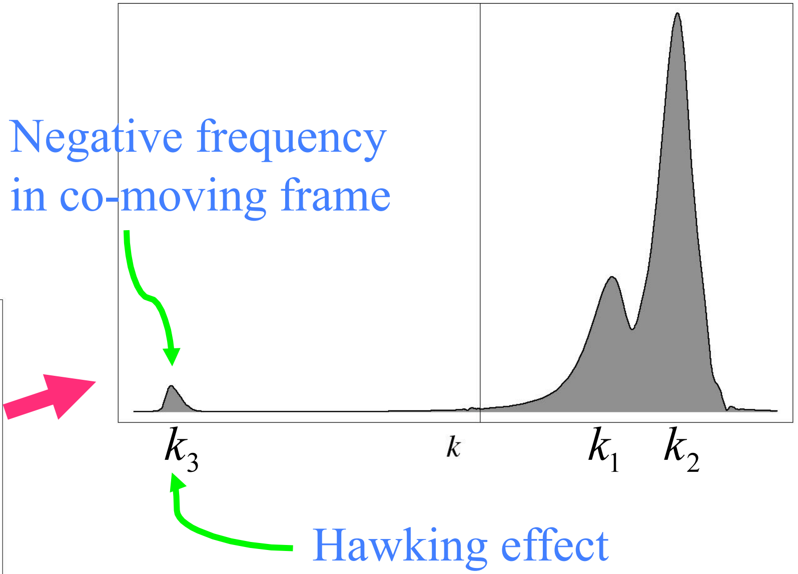
$\omega$ - Frequency in lab frame (conserved)





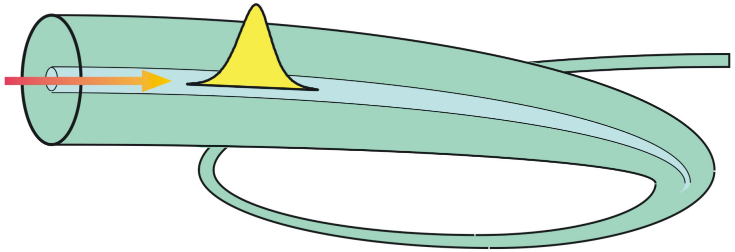



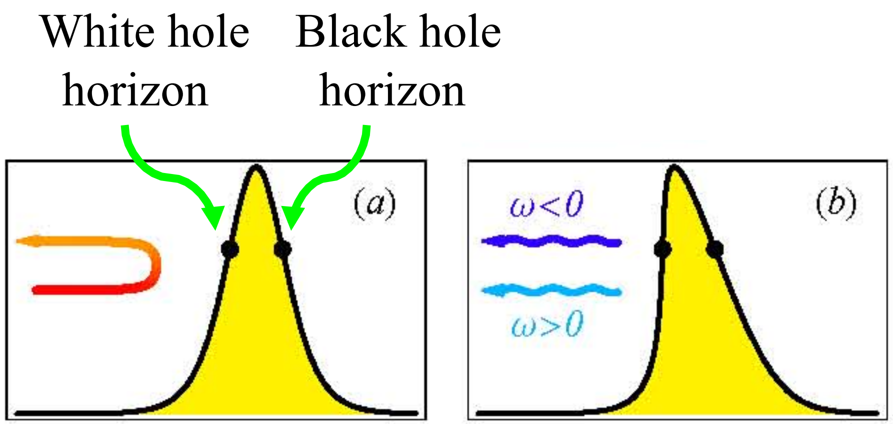
# Horizons in optical fibres

Due to Kerr effect, a pulse in an optical fiber creates a change in refractive index that moves at the speed of light: effective moving medium.

A weak probe beam can be slowed to a standstill in the frame of the pulse – a horizon.

$$n = n_0 + \delta n, \quad \delta n \propto I(z,t).$$






$\tau = t - \frac{z}{u}, \quad \zeta = \frac{z}{u}$ $u$ – speed of pulse

As before, dispersion (normal) limits blue-shifting.

Horizon mixes positive and negative lab frequencies – Hawking Effect. Driven by steepness of pulse (“surface gravity”) at horizon. White-hole horizon radiates more because of pulse self-steepening.

Equation for probe:

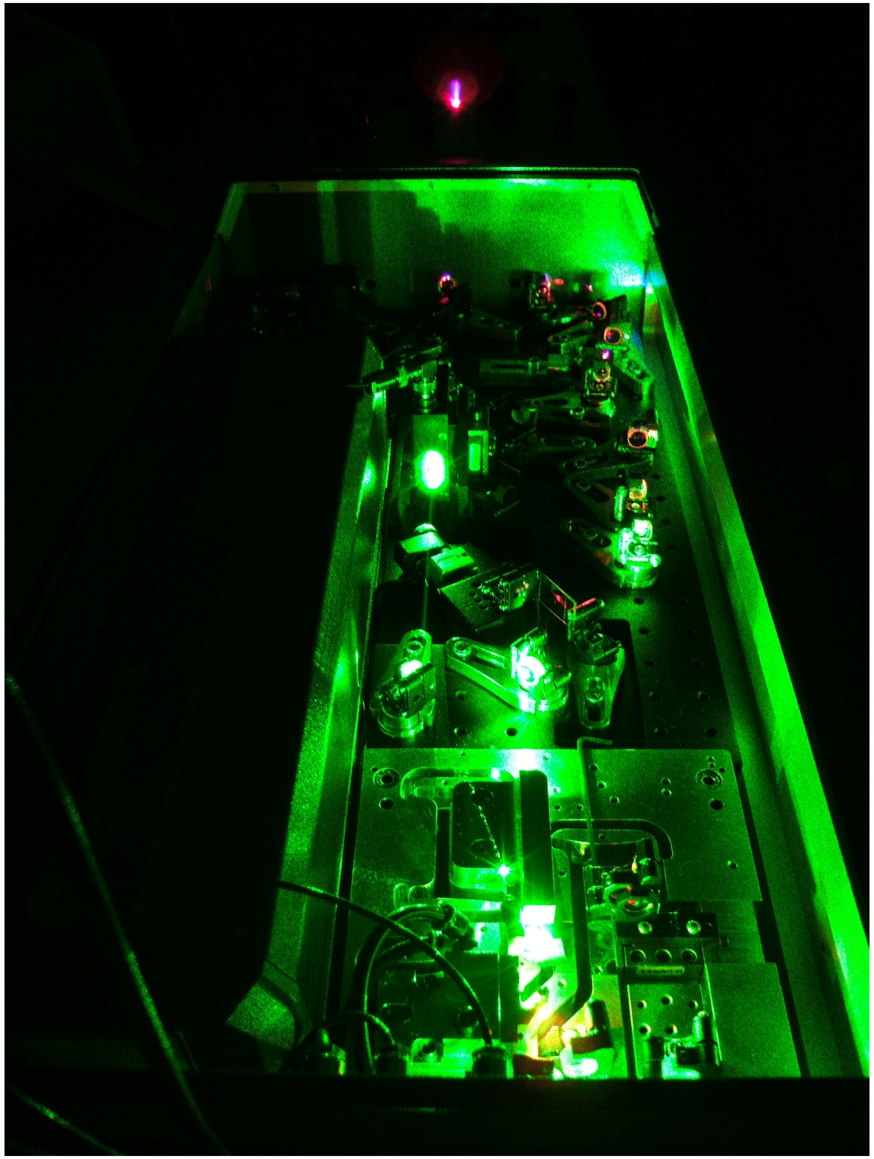
$$(\partial_\zeta - \partial_\tau)^2 A = \partial_\tau \frac{u^2}{c^2} n^2 \partial_\tau A$$

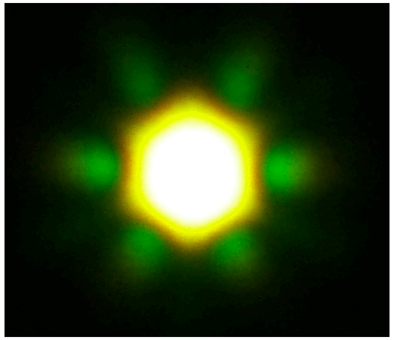
Dispersion relation: $\omega' = \left(1 - \frac{n(\omega)u}{c}\right)\omega.$

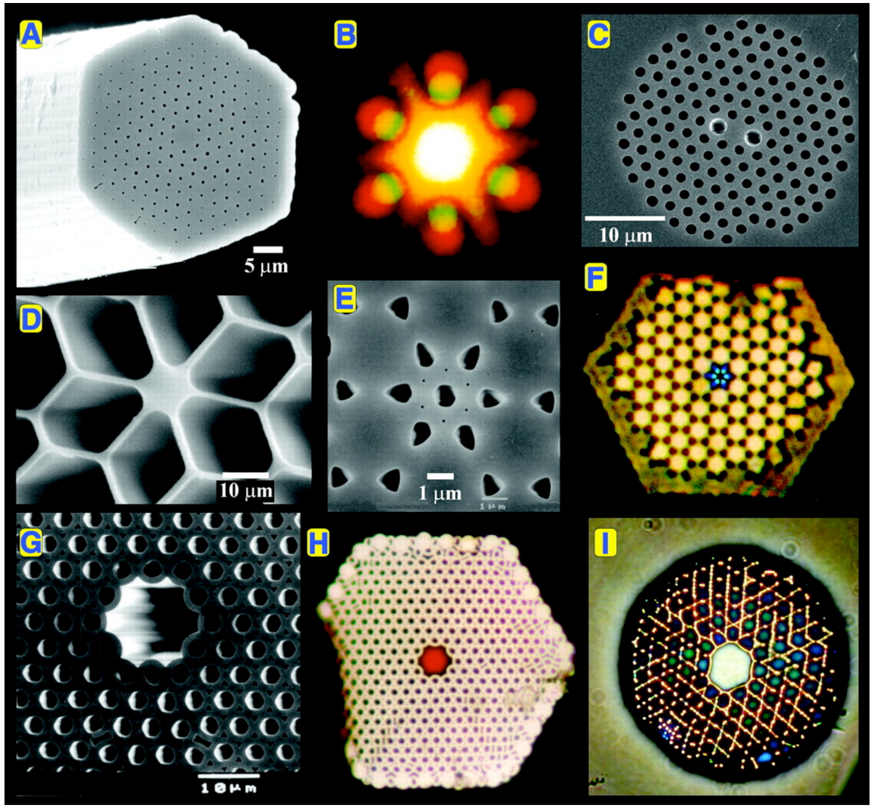
Frequency in pulse frame (conserved)

Frequency in lab frame

# Few-cycle pulses in microstructured fibres

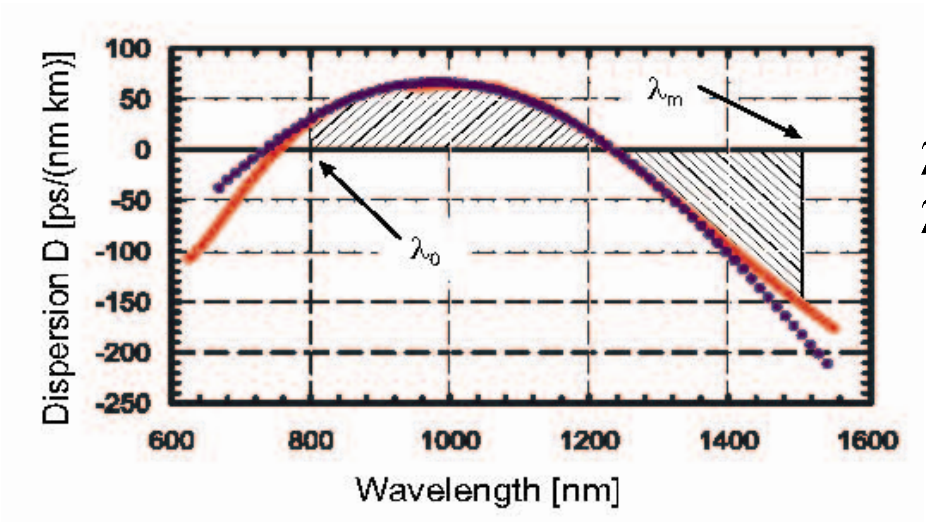






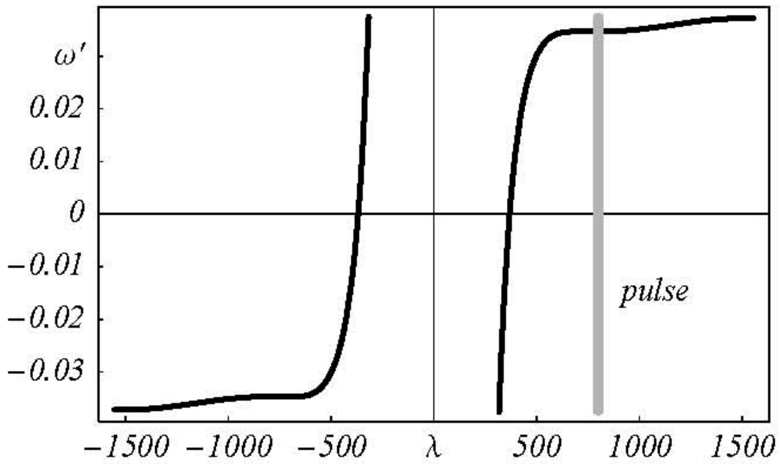
# Frequency-shifting at the horizons

Choose probe wavelength such that a group-velocity horizon exists.



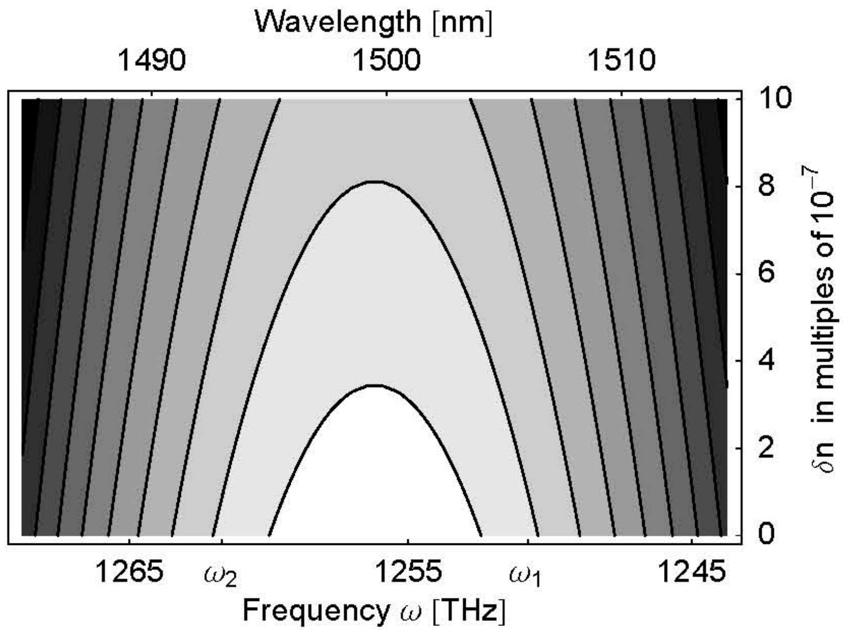

λ0 – Pulse carrier wavelength.

λm – Group-velocity-matched probe wavelength.




$$\omega' = \left(1 - \frac{n(\omega)u}{c}\right)\omega.$$

$\omega'$ is conserved during probe evolution.

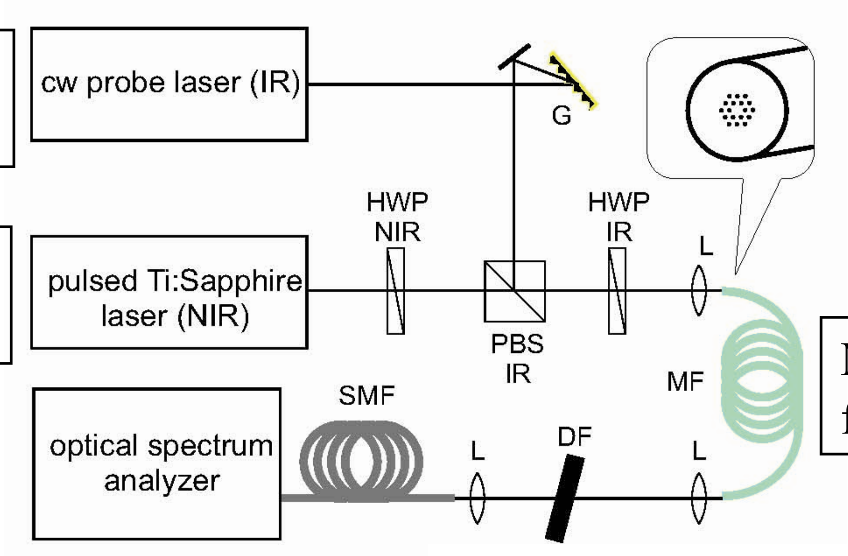



Contour lines of $\omega'$. Frequency of probe follows contour line. Blue/red shifting at white/black-hole horizon.

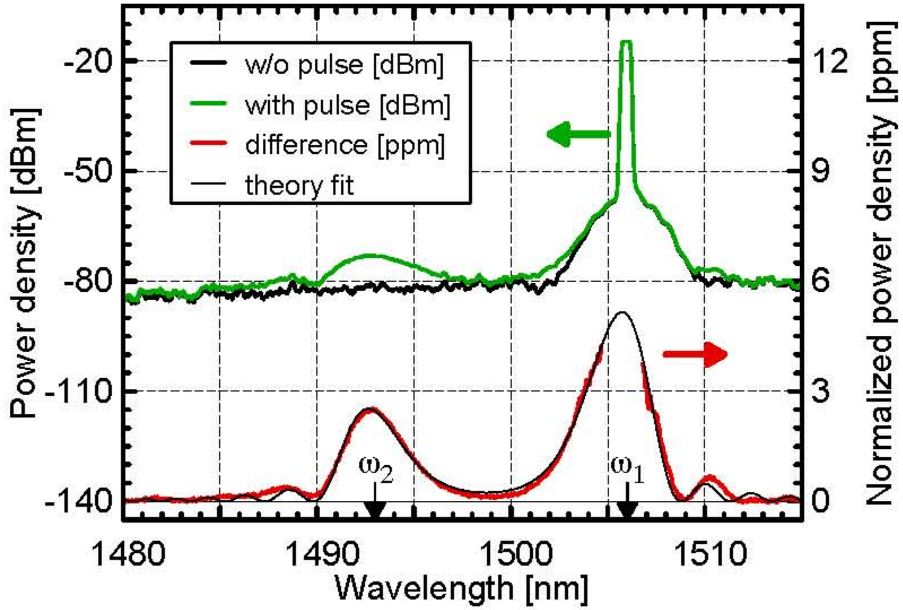
# Observation of blue-shifting at the white-hole horizon

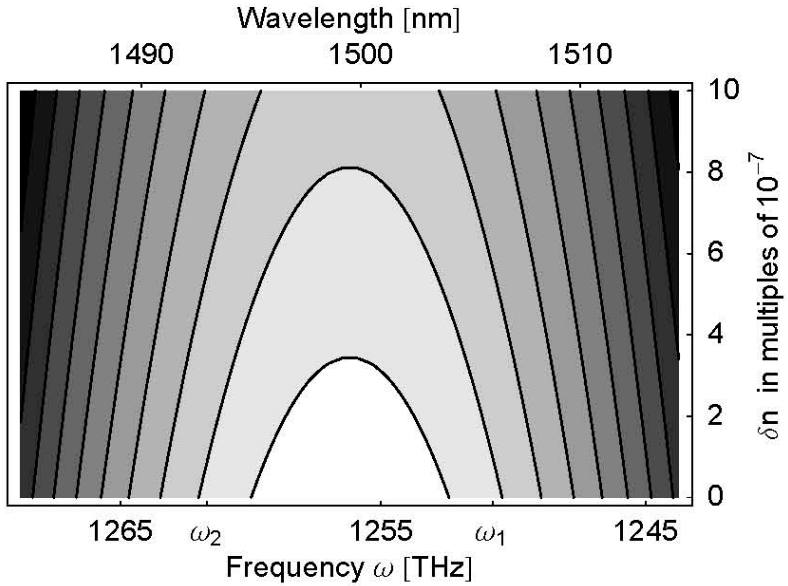
CW diode laser,
tunable 1460 to 1540nm

70fs (FWHM) pulses
(≈800nm) at 80MHz



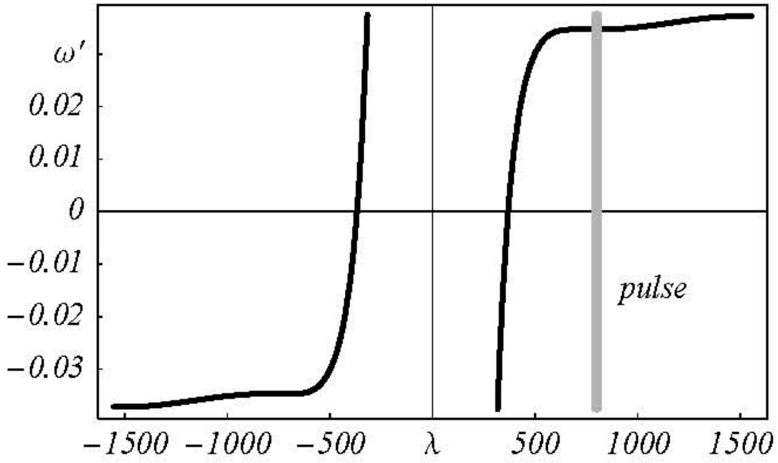

Microstructured
fibre



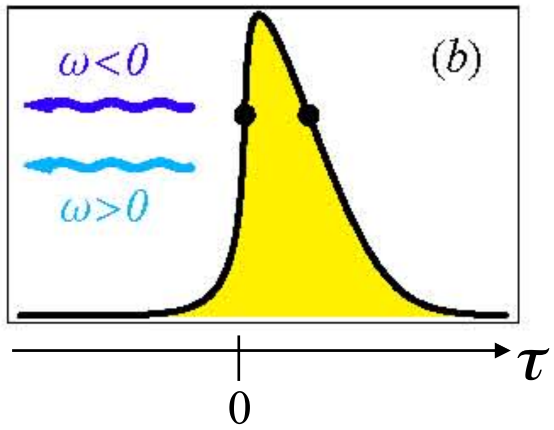

[Philbin *et al.*, *Science* **319**, 1367 (2008)]

# Hawking radiation in an optical fiber?




$$n = n_0 + \delta n$$

$$\omega' = \left(1 - \frac{n(\omega)u}{c}\right)\omega$$




$\omega' = 0$ in near UV (≈300nm) where phase velocity *c/n* matches group velocity *u* of pulse – a phase-velocity horizon. Each $\omega'$ gives two lab frequencies, one positive and one negative, which are mixed by the horizon. Creation of correlated pairs of photons.

Linearize $\delta n$ around phase-velocity horizon $1 - \frac{nu}{c} = \alpha'\tau$

Result: thermal spectrum in pulse frame $k_B T' = \frac{\hbar\alpha'}{2\pi}$

Temperature in lab frame: $k_B T = \frac{\hbar\alpha}{2\pi}, \quad \alpha = -\frac{1}{\delta n}\frac{\partial\,\delta n}{\partial\tau}\Big|_{\tau=0}$

*T* does not depend on the magnitude of (very small) $\delta n$.

If steepness at horizon is ~pulse carrier frequency, then *T* ~ 1000K.

# Horizon physics: astrophysics, fluid mechanics, optics…