UNIVERSITY OF MARYLAND

Highly nonlinear fluid flows in water, helium, glycerin, and sodium

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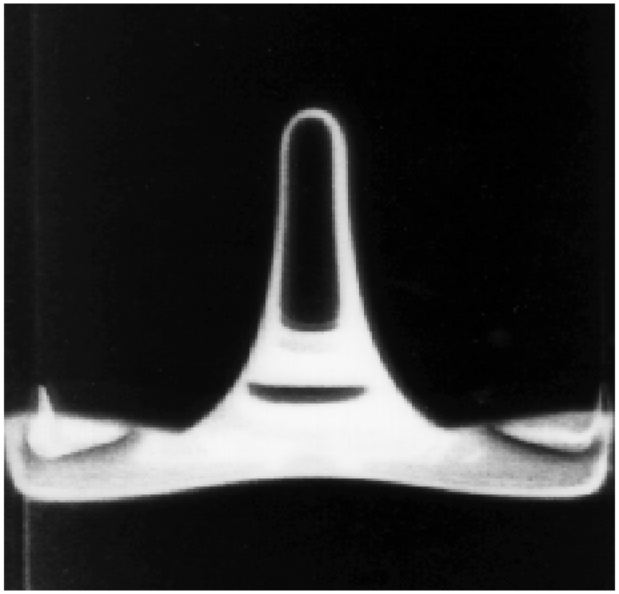
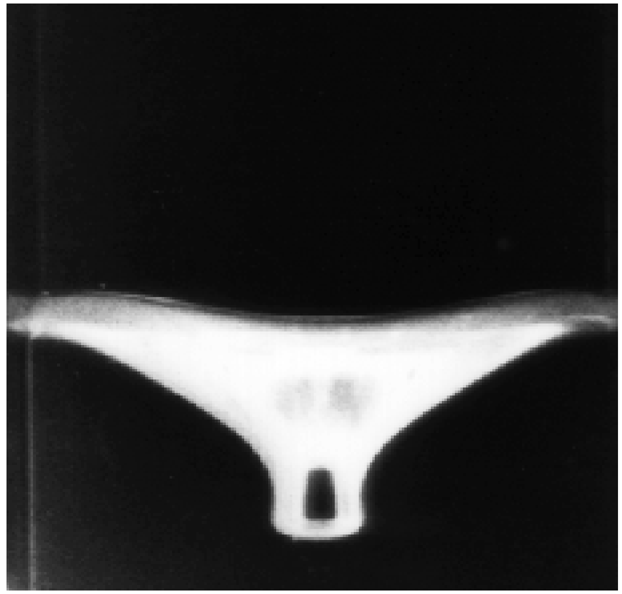
http://complex.umd.edu

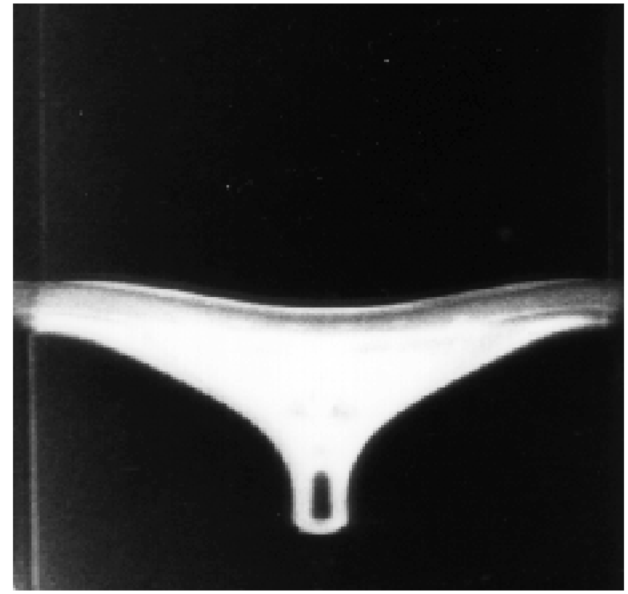
1) Surface wave jets and turbulent intermittency
2) Rapidly rotating turbulent flows



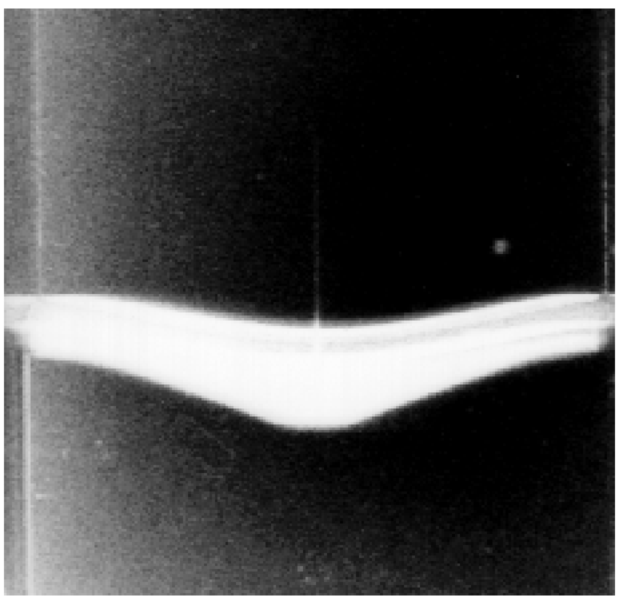
−0.156s



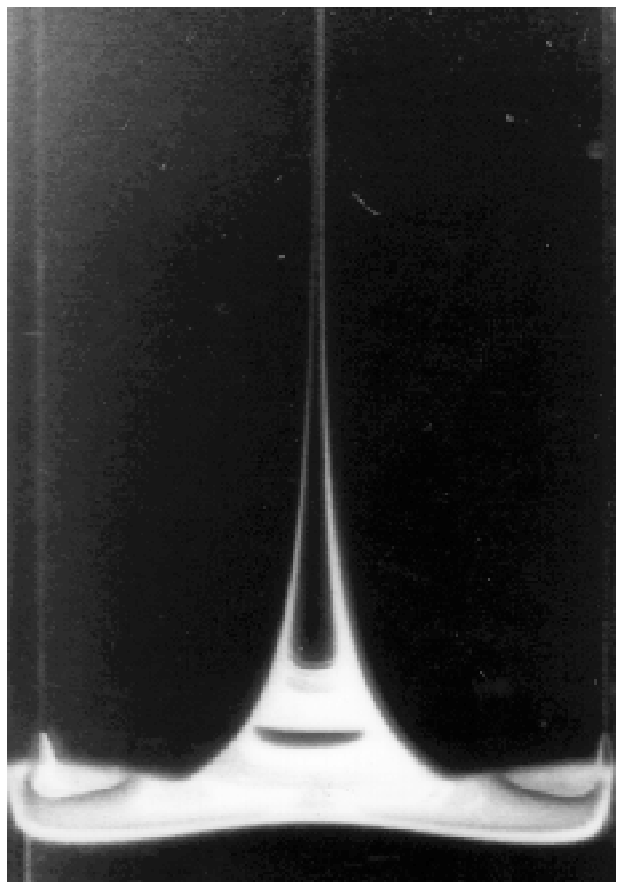
−0.016s



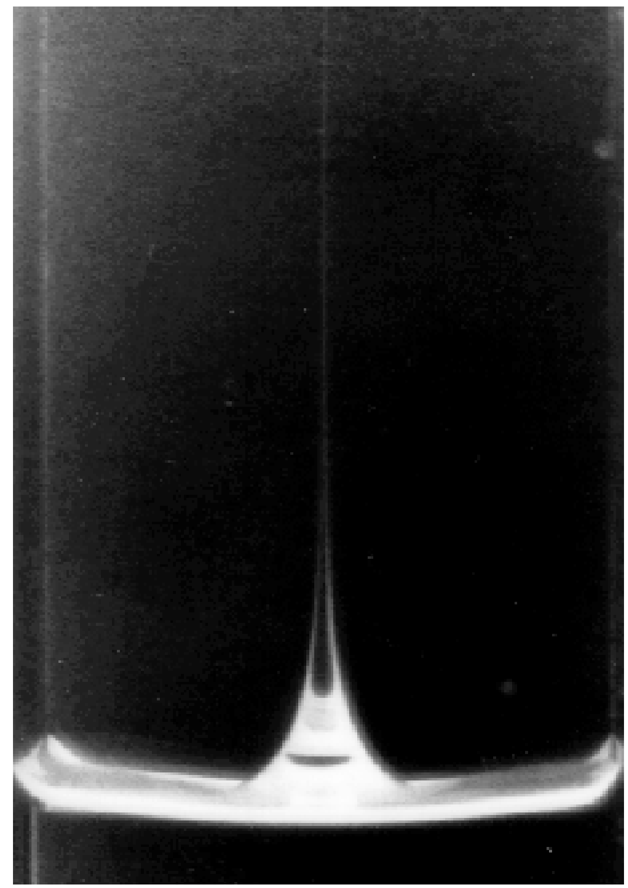
−0.008s



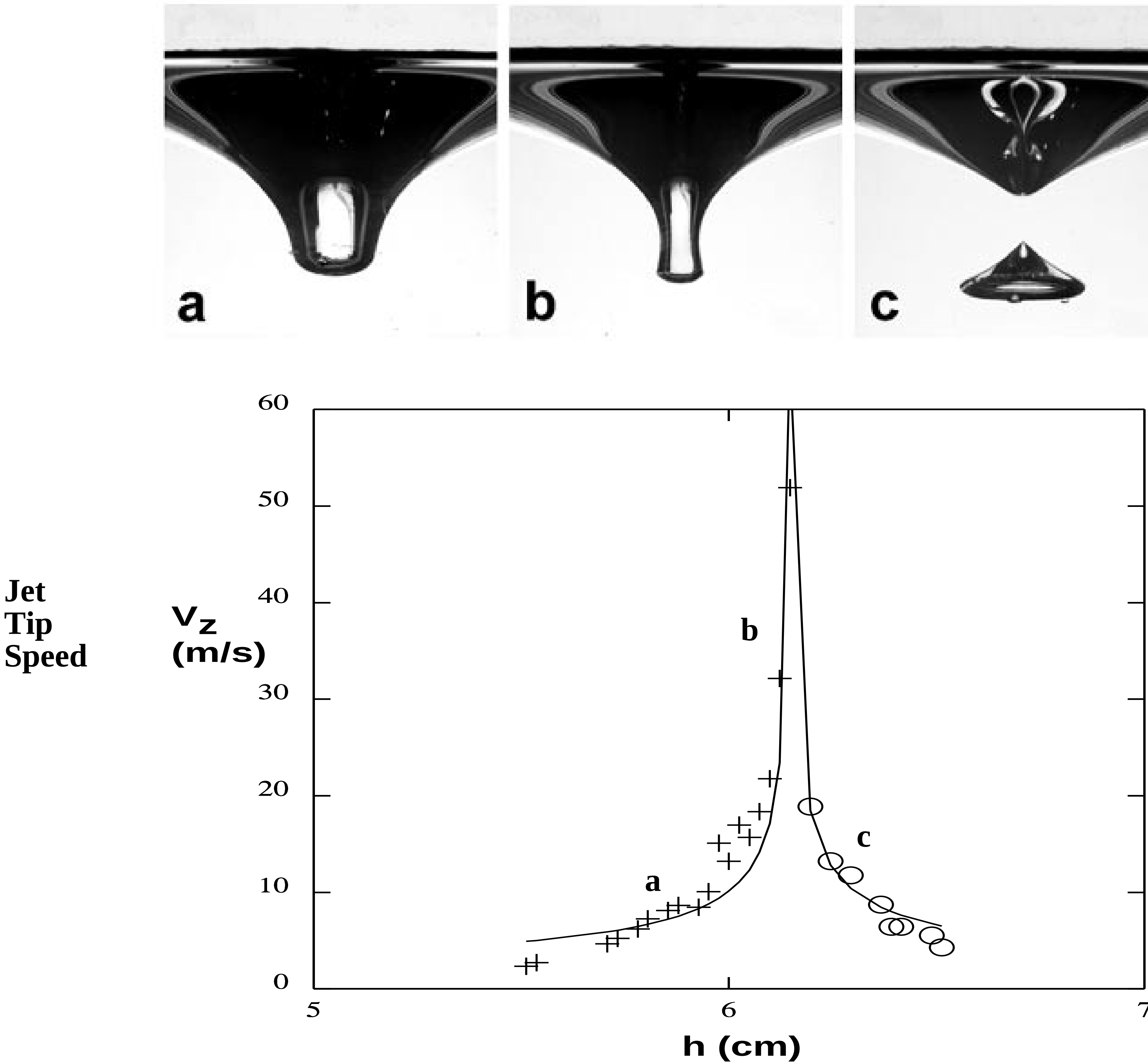
0.004s



0.044s

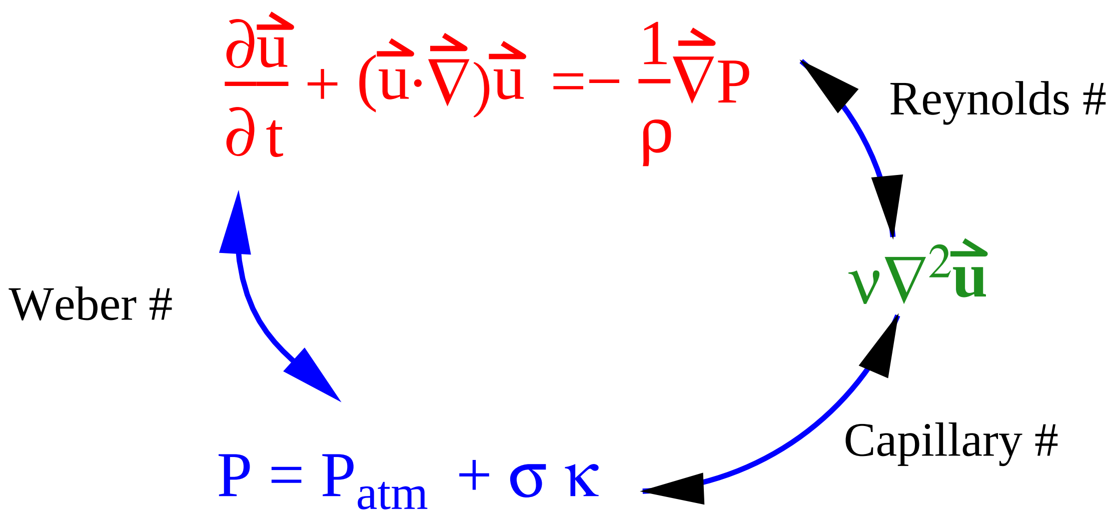


0.084s



a
b
c
Jet
Tip
Speed
vz
(m/s)
60
50
40
30
20
10
0
5
6
7
h (cm)
a
b
c

Force balance leads to 3 scaling regimes



$l \sim |\tau|^{\alpha}$

Navier−Stokes (i−ν)

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\vec{\nabla})\vec{u} = -\frac{1}{\rho}\vec{\nabla}P + \nu\nabla^2\vec{\mathbf{u}}$$

$l \sim |\tau|^{1/2}$

Bernoulli (i−σ)

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}(\vec{\nabla}\phi)^2 + \frac{\sigma}{\rho}\kappa = 0$$

$l \sim |\tau|^{2/3}$

Stokes (ν−σ)

$$\nu\nabla^2\vec{u} = \frac{1}{\rho}\vec{\nabla}P$$

$$P = P_{atm} + \sigma\,\kappa + \text{visc. terms}$$

$l \sim |\tau|$

Similarity Equations

$\alpha = 2/3 \quad \gamma = 1/3$

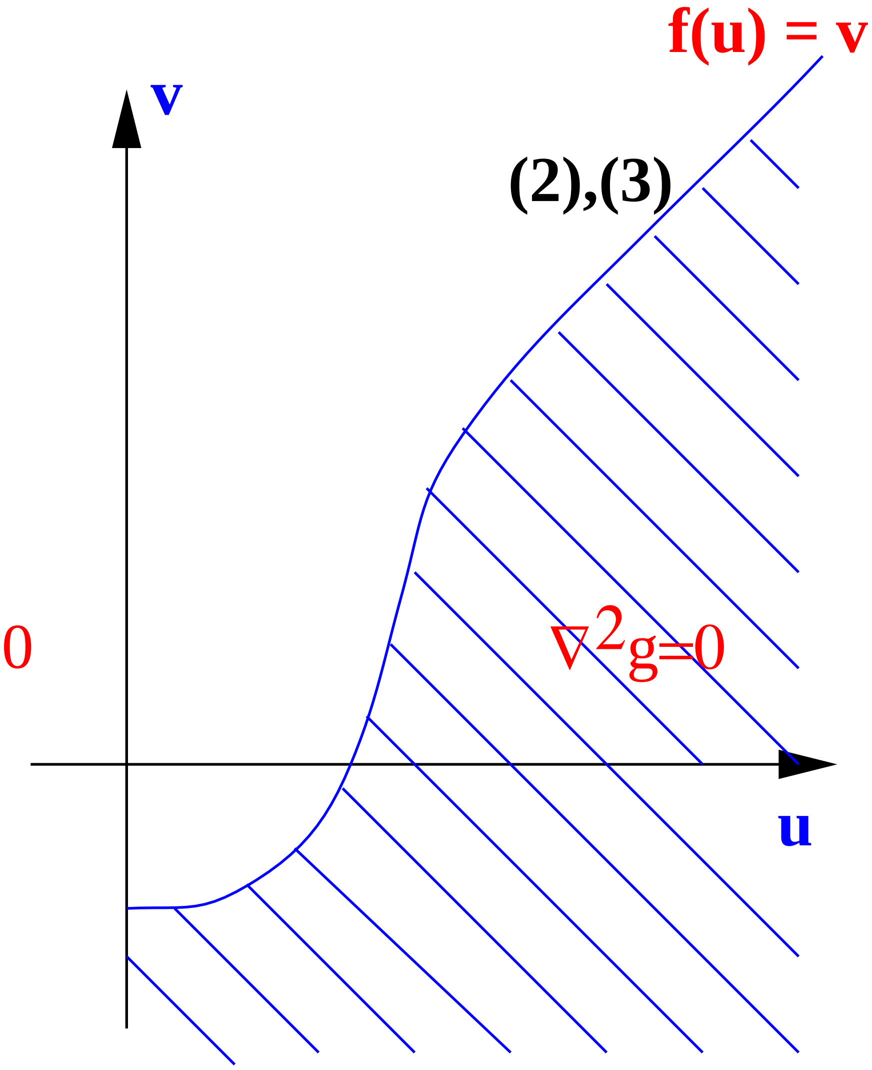
Time drops out of problem

$$\mathbf{u = r / (-t)^{2/3}}$$
$$\mathbf{f(u) = v = z / (-t)^{2/3}}$$
$$\mathbf{g(u,v) = (-t)^{1/3}\, \phi(r,z,t)}$$

(1) $\nabla^2 g = 0$

(2) $\dfrac{\partial f}{\partial u} = \dfrac{g_v - 2f/3}{g_u - 2u/3}$

(3) $-\dfrac{g}{3} + \dfrac{2u}{3} g_u + \dfrac{2f}{3} g_v + \dfrac{1}{2}(\nabla g)^2 + \kappa = 0$



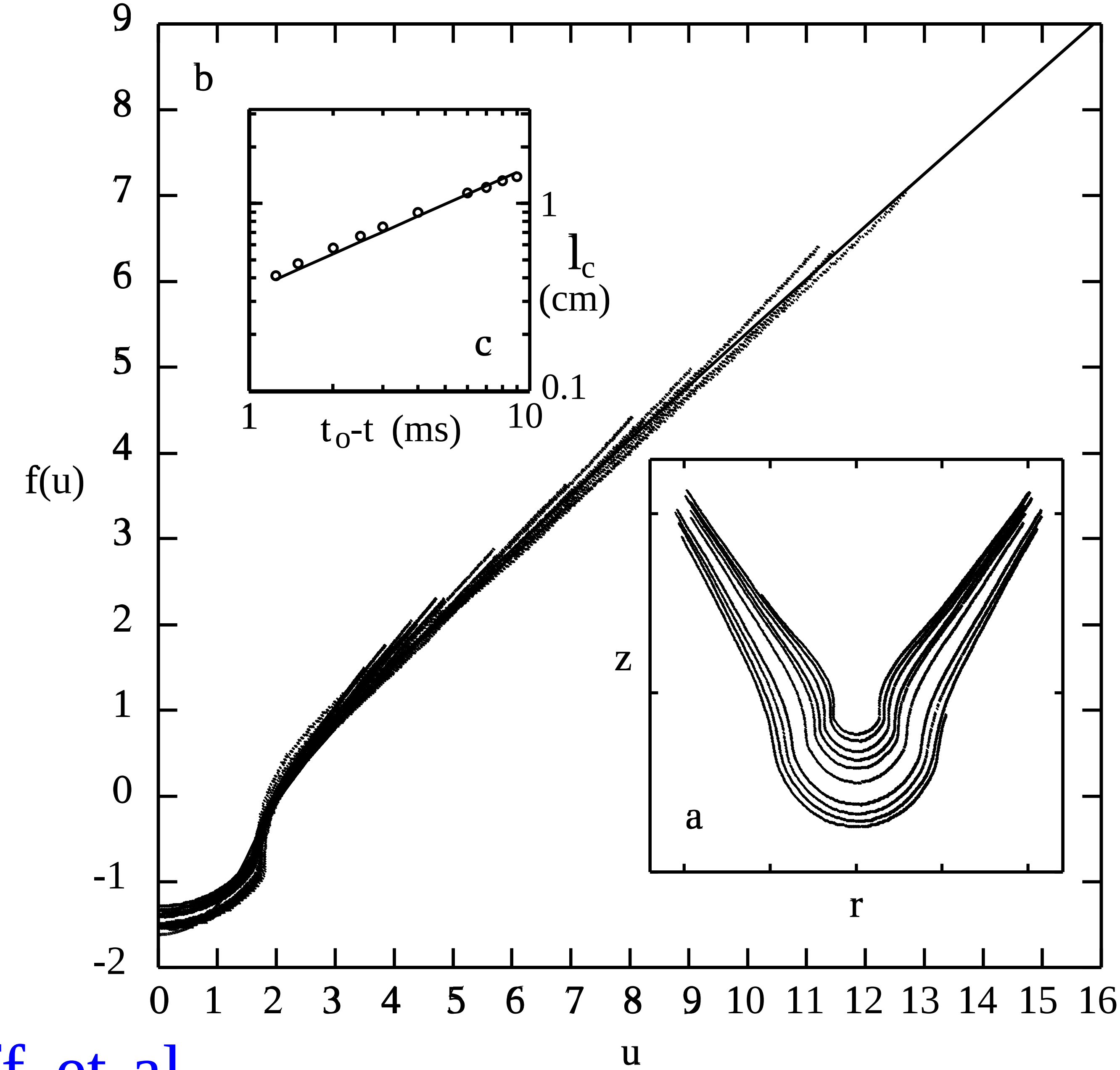
Asymptotics

large radius --> cone

$f \sim u$ angle free

$g \sim u^{1/2}$

small radius --> regular

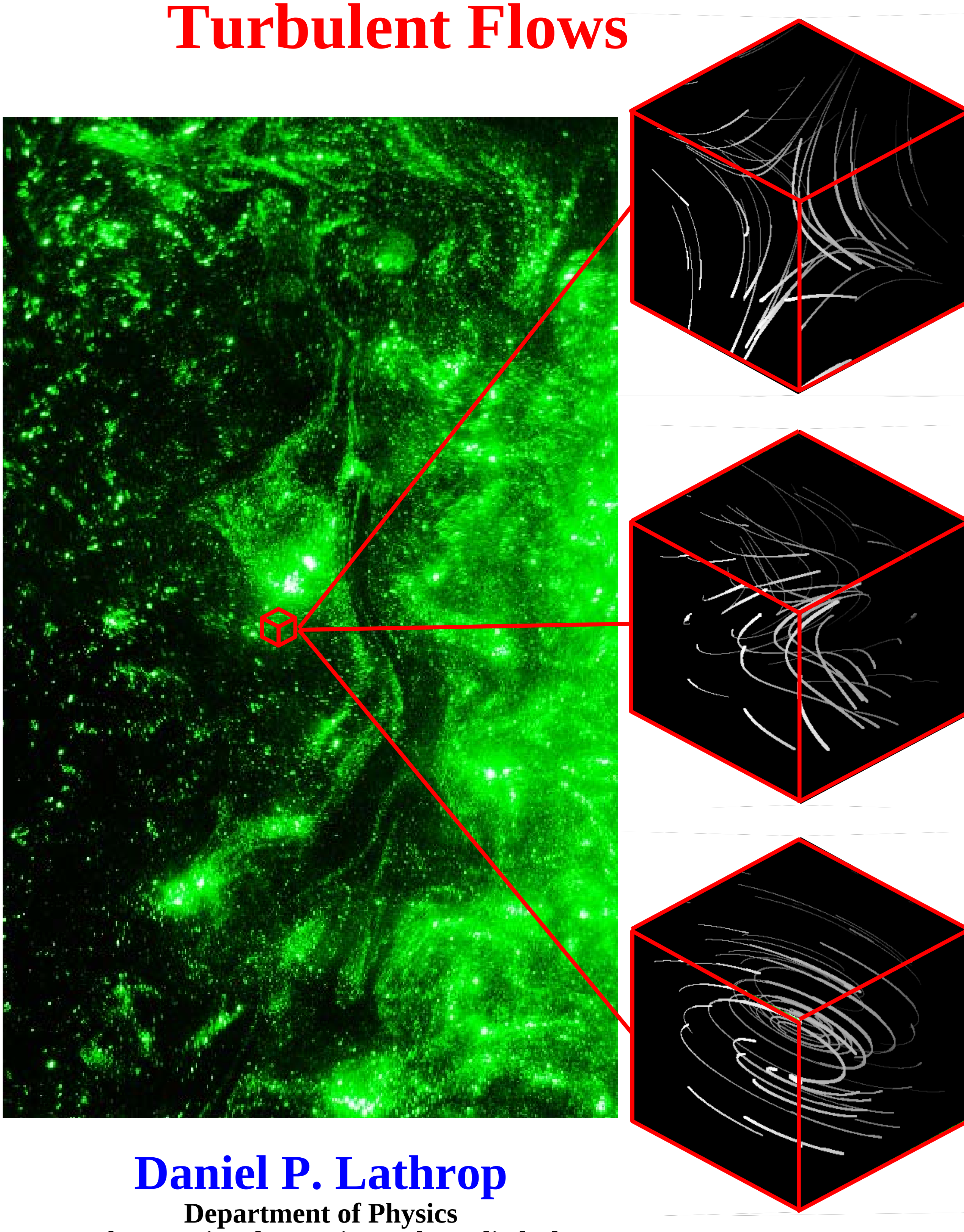


$$f(u) = z/\tau^{2/3}$$

$$u = r/\tau^{2/3}$$

B.W. Zeff, et al.
Nature 403, 401, (2000)

Intense Rotation and Dissipation in Turbulent Flows



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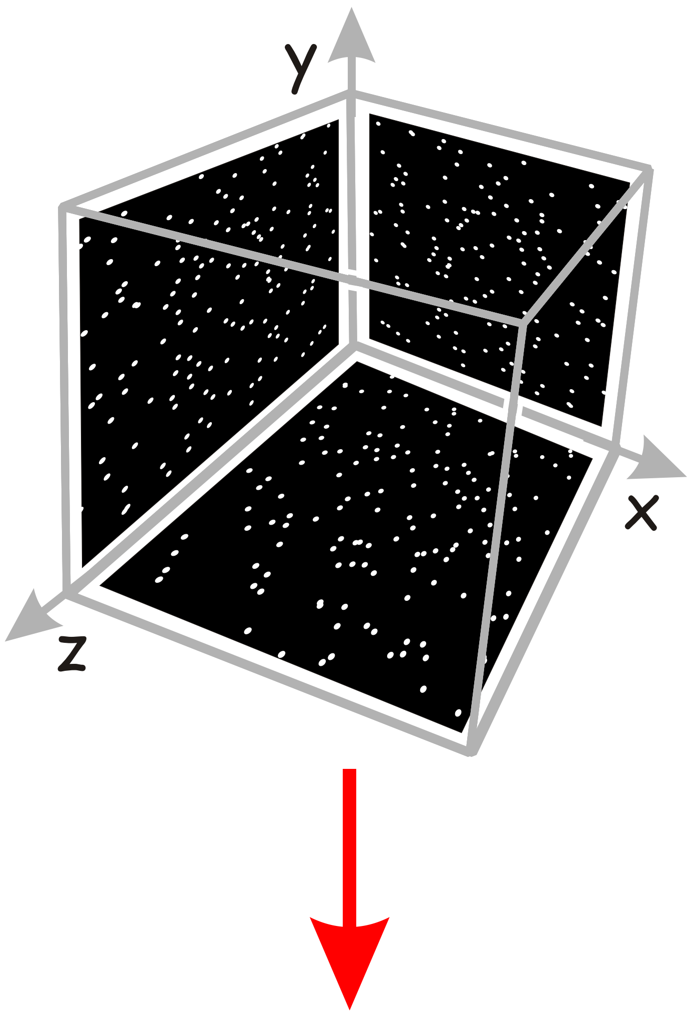
At Arizona State University: Eric Kostelich
Funding: NSF−DMR

Measuring Gradients



3 camera views

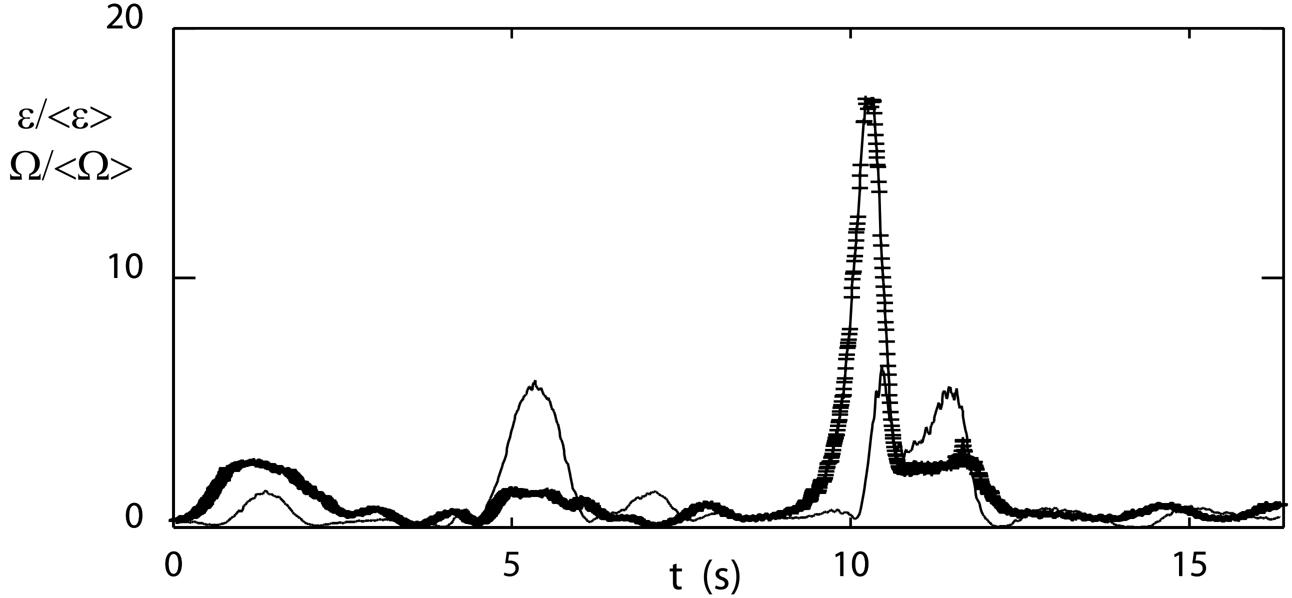
fit data to model

$$\vec{v} = \vec{v}_o + M \cdot \vec{x}$$

$$M = \begin{bmatrix} \partial_x V_x & \partial_y V_x & \partial_z V_x \\ \partial_x V_y & \partial_y V_y & \partial_z V_y \\ \partial_x V_z & \partial_y V_z & \partial_z V_z \end{bmatrix}$$

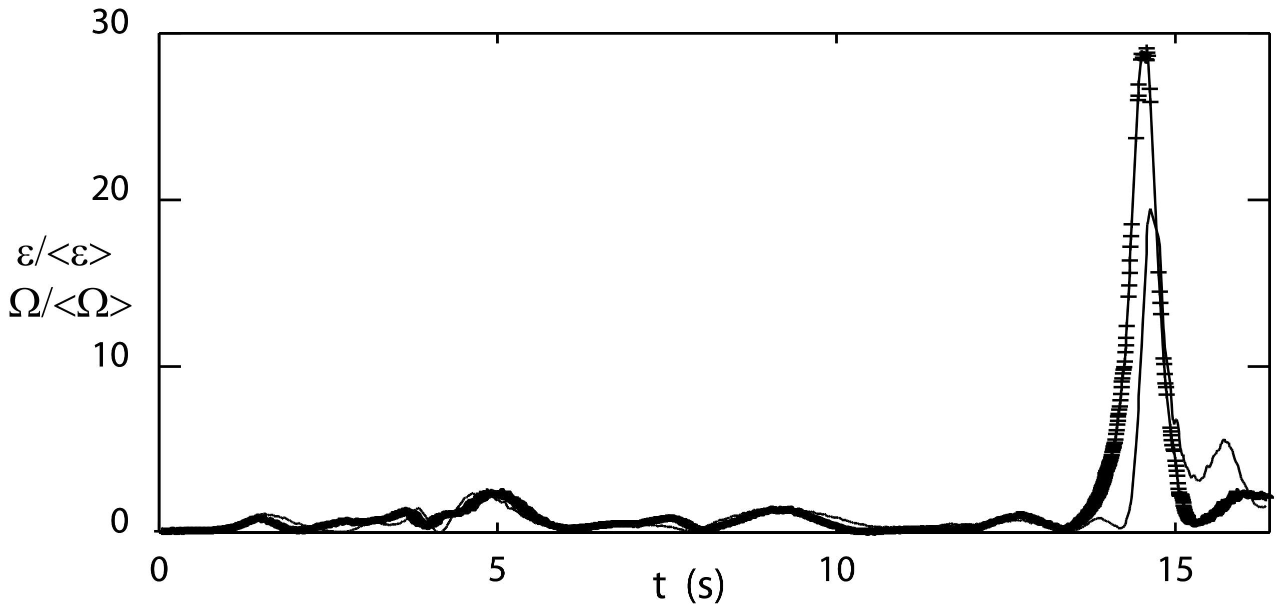
Dissipation and Enstrophy Time Series -- Intense Events

Zeff, Lanterman, McAllister, Roy, Kostelich, and Lathrop, Nature 421, 146 (2003)



$$\varepsilon = \nu / 2 \, \| M_{ij} + M_{ji} \|_2$$

$$\Omega = 2 \, \| M_{ij} - M_{ji} \|_2 = \omega^2/2$$



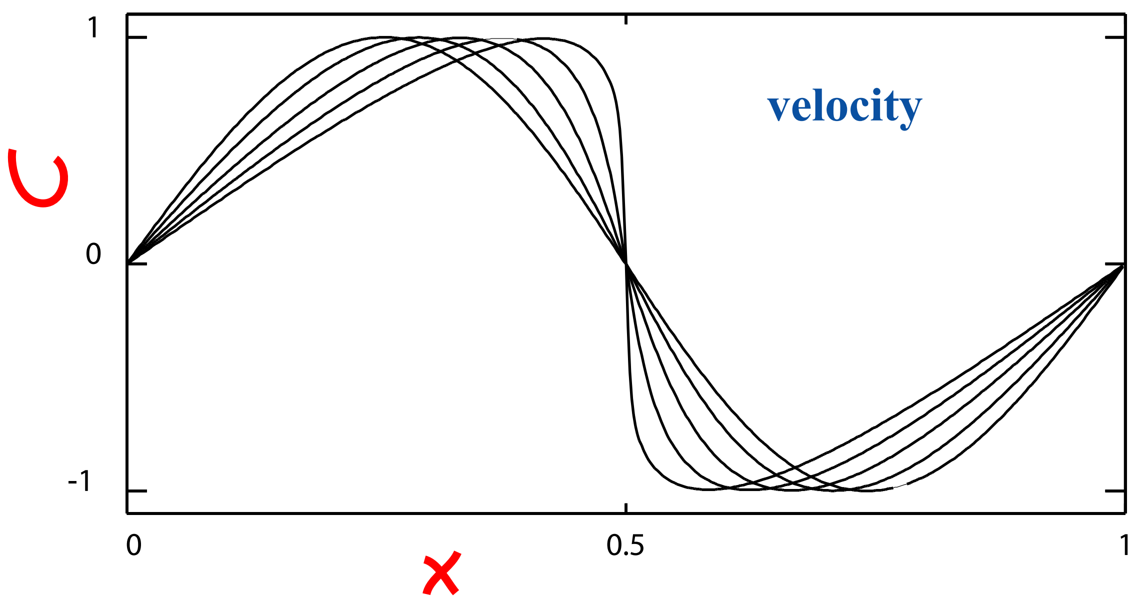
Burgers Equation

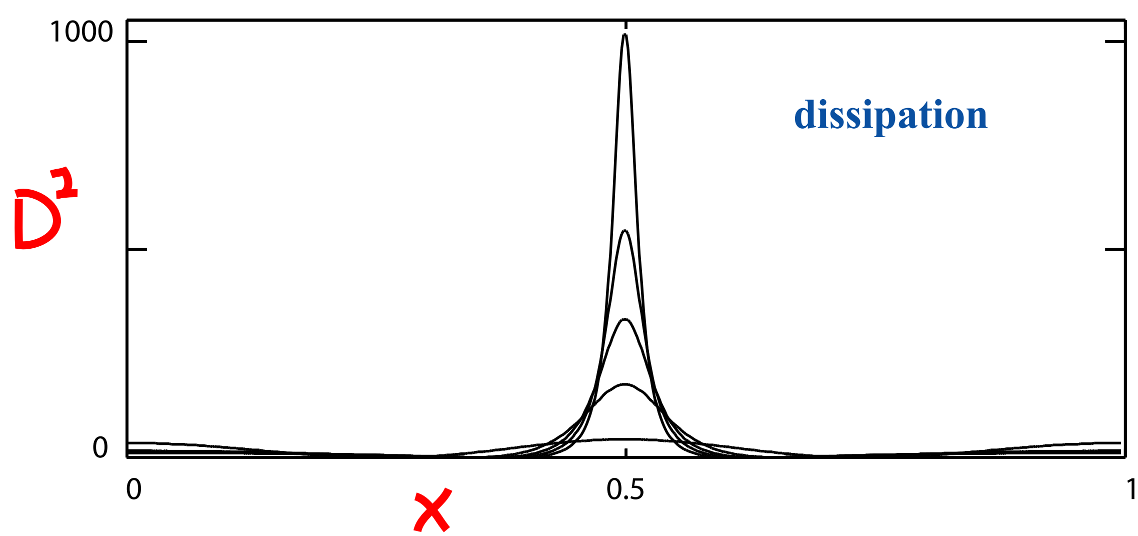
Gradient steepening

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2}$$

$$D \equiv \frac{\partial u}{\partial x}$$

$$\frac{\partial D}{\partial t} + u\frac{\partial D}{\partial x} = \nu\frac{\partial^2 D}{\partial x^2} - D^2$$





Euler Equation

$$\frac{\partial \underline{u}}{\partial t} + (\underline{u}\cdot\underline{\nabla})\underline{u} = -\frac{1}{\rho}\underline{\nabla} P$$

$$\underline{\nabla}\cdot\underline{u}=0$$

$$\partial_i u_j = \tau^{-1}\, \partial'_i G_j \quad \text{at origin}$$

Ansatz

$$l \sim \tau^{\alpha}$$

$$v \sim \tau^{\gamma}$$

$$\underline{v}(\underline{x},t) = \tau^{\gamma}\, \underline{G}(\, \underline{x} / \tau^{\alpha})$$

$\alpha < 1$ local

$\alpha = 2/5$ Constantin, Green, Pelz

$\alpha = 3/2$ corresp. K41

$$\underline{G} = r^{\gamma/\alpha}\, \underline{f}\,(\theta,\phi)$$

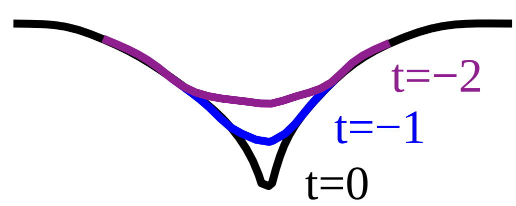
Similarity equations (time independent)

arXiv:cond-mat/0311487

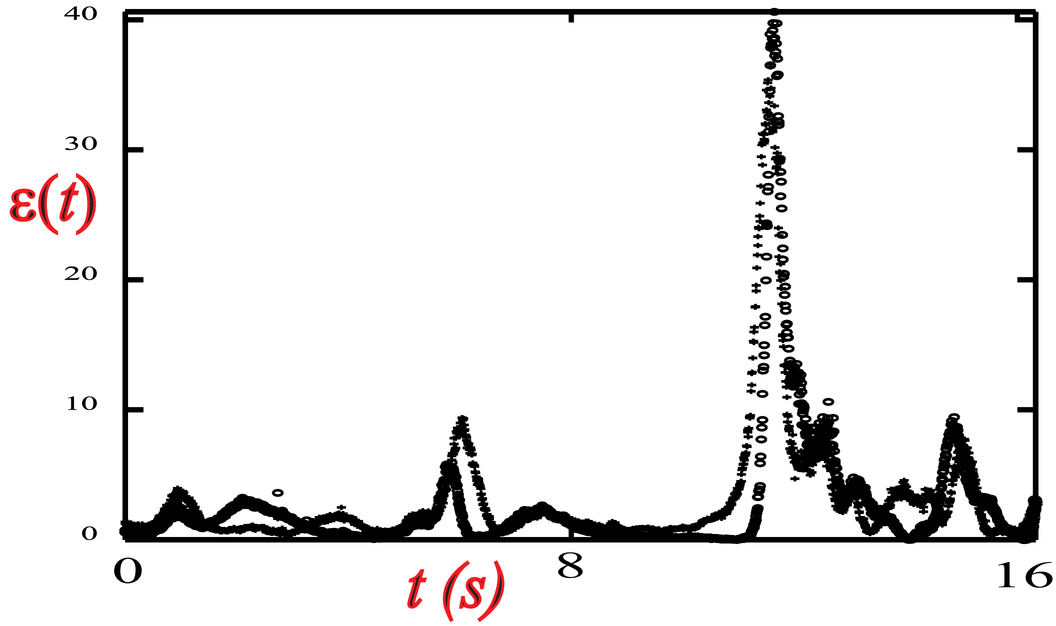
$$-\gamma\, \underline{G} + \alpha(\underline{r}\cdot\underline{\nabla}')\underline{G} + (\underline{G}\cdot\underline{\nabla}')\underline{G} + \underline{\nabla}'\Pi = 0$$

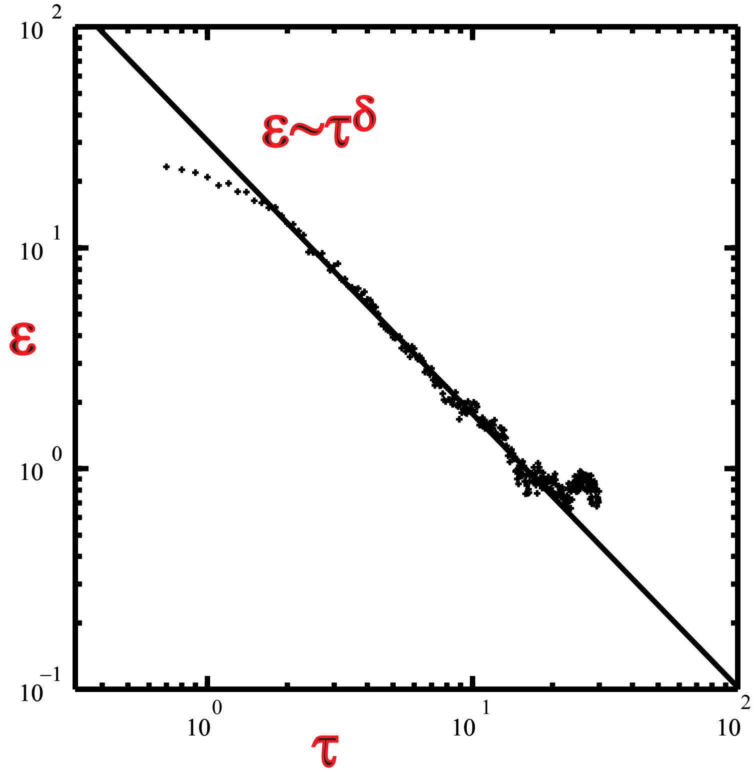
$$\underline{\nabla}'\cdot\underline{G}=0$$

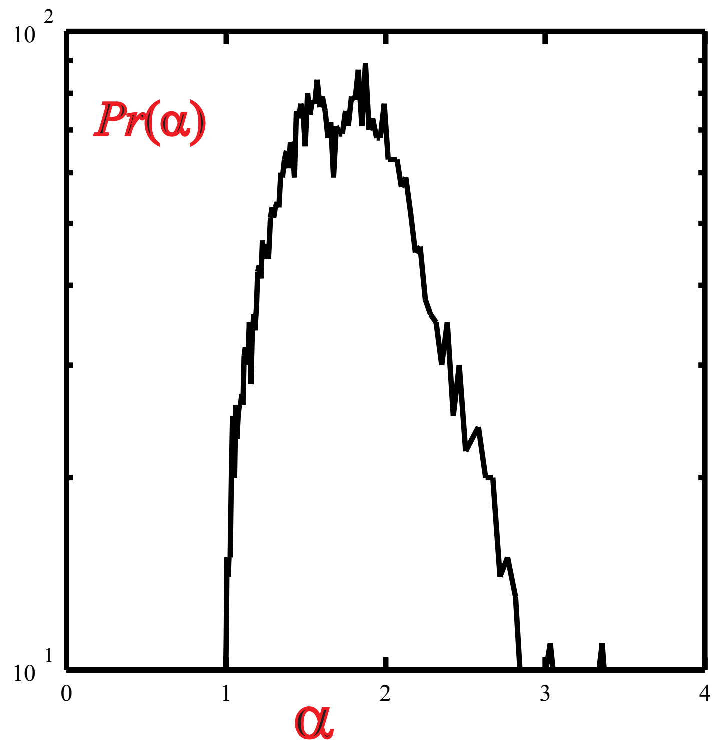
$$\gamma = \alpha - 1$$



Time dependence of dissipation rise







$$\alpha = -2/\delta$$

Rapidly Rotating -- Coriolis Large

$$\partial_t \vec{v} + \left(\vec{v} \bullet \vec{\nabla}\right) \vec{v} + 2\vec{\Omega} \times \vec{v} = -\frac{1}{\rho}\vec{\nabla} P + \nu \nabla^2 \vec{v}$$

$$\vec{\nabla} \bullet \vec{v} = 0$$

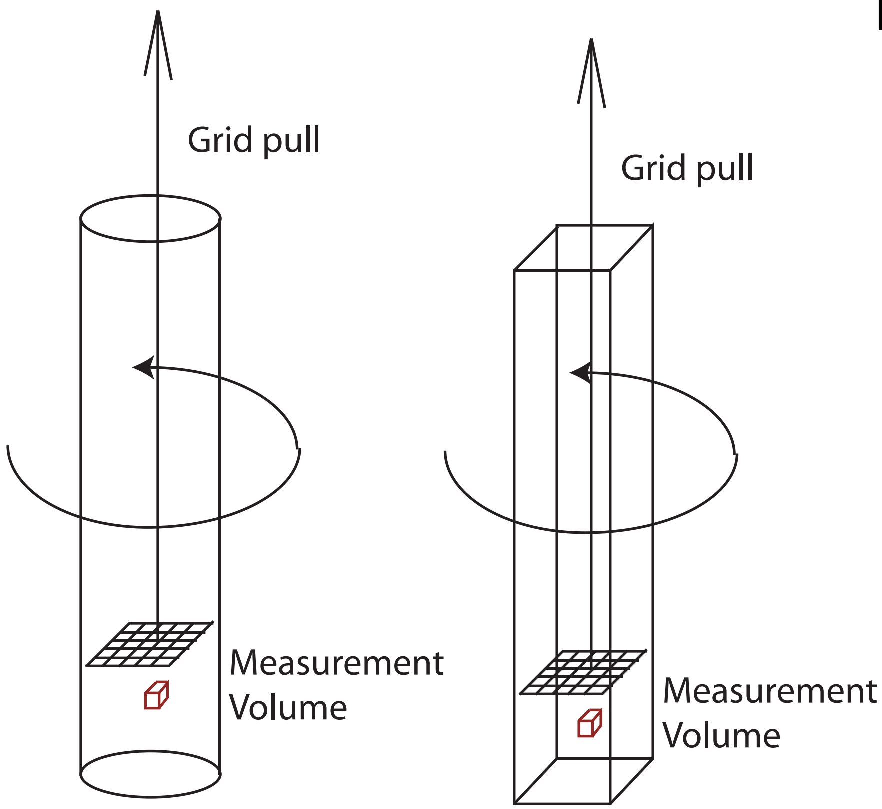
$$\partial_t \vec{v} + 2\vec{\Omega} \times \vec{v} = -\frac{1}{\rho}\vec{\nabla} P$$

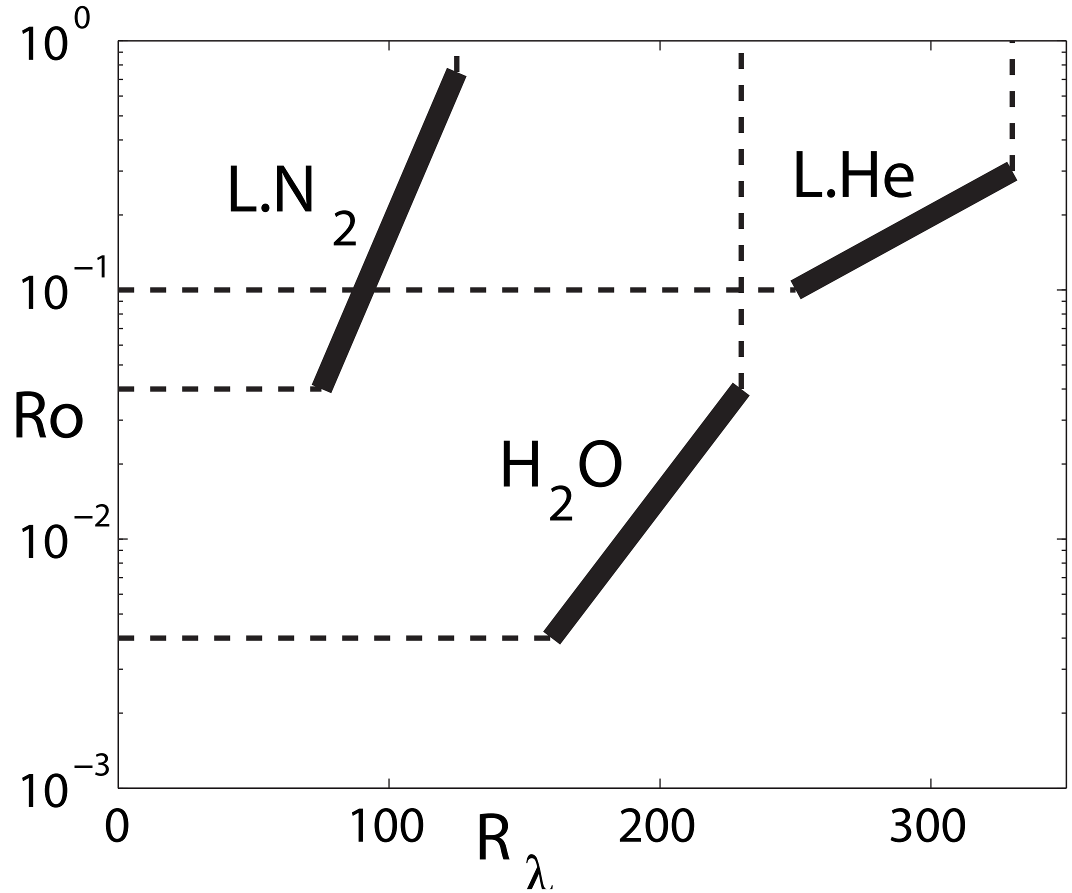
$$2\vec{\Omega} \times \vec{v} = -\frac{1}{\rho}\vec{\nabla} P$$

$$(\vec{\Omega} \bullet \vec{\nabla})\vec{v} = 0$$

Taylor-Proudman theorem

Grid generated turbulence





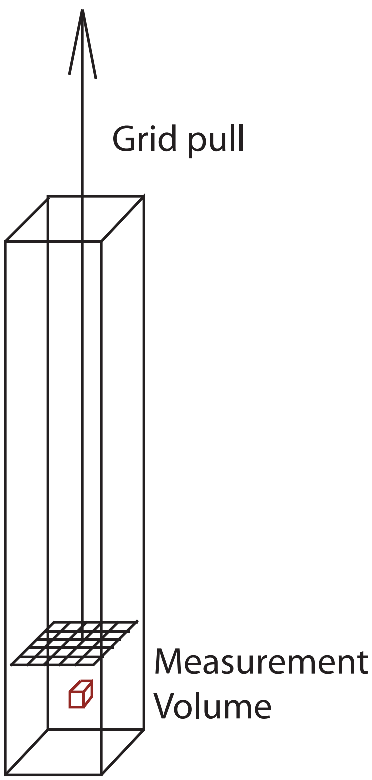
$Ro = v_{rms} / \Omega b$ Rossby No.

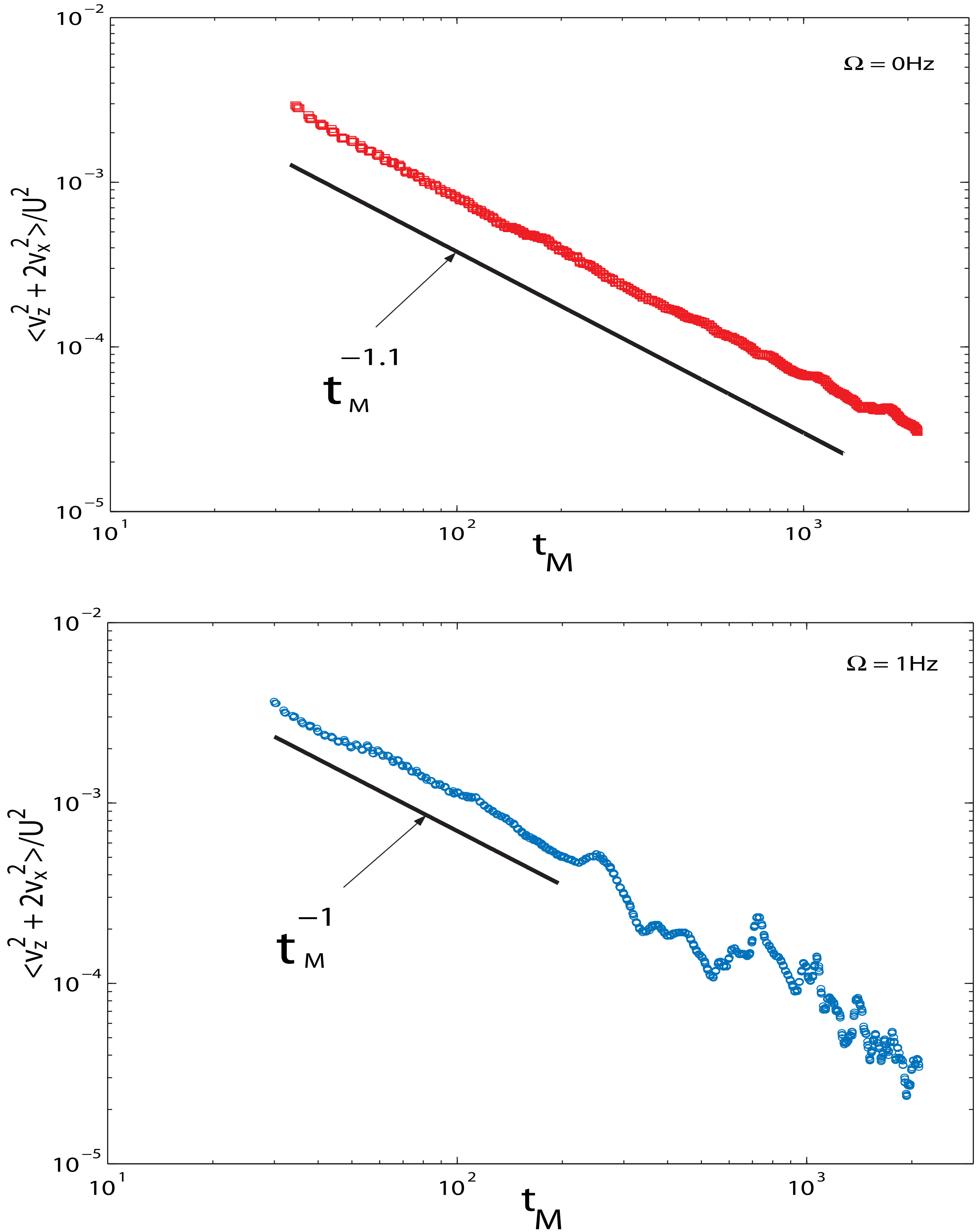
$R_\lambda = v_{rms}\, \lambda / \nu$ Reynolds No.

Decay of kinetic energy

Liquid Nitrogen

Grid generated turbulence

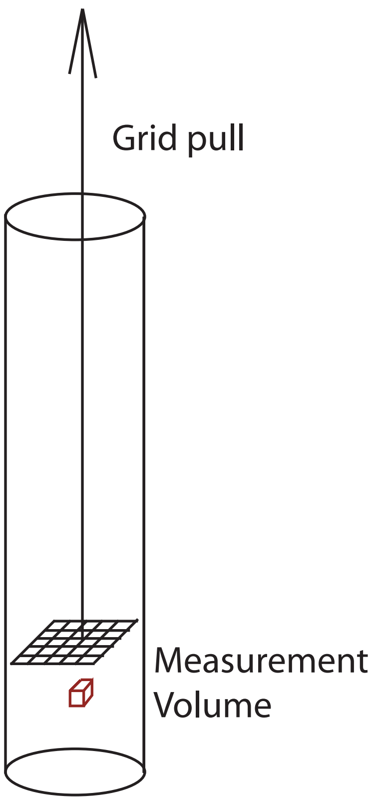


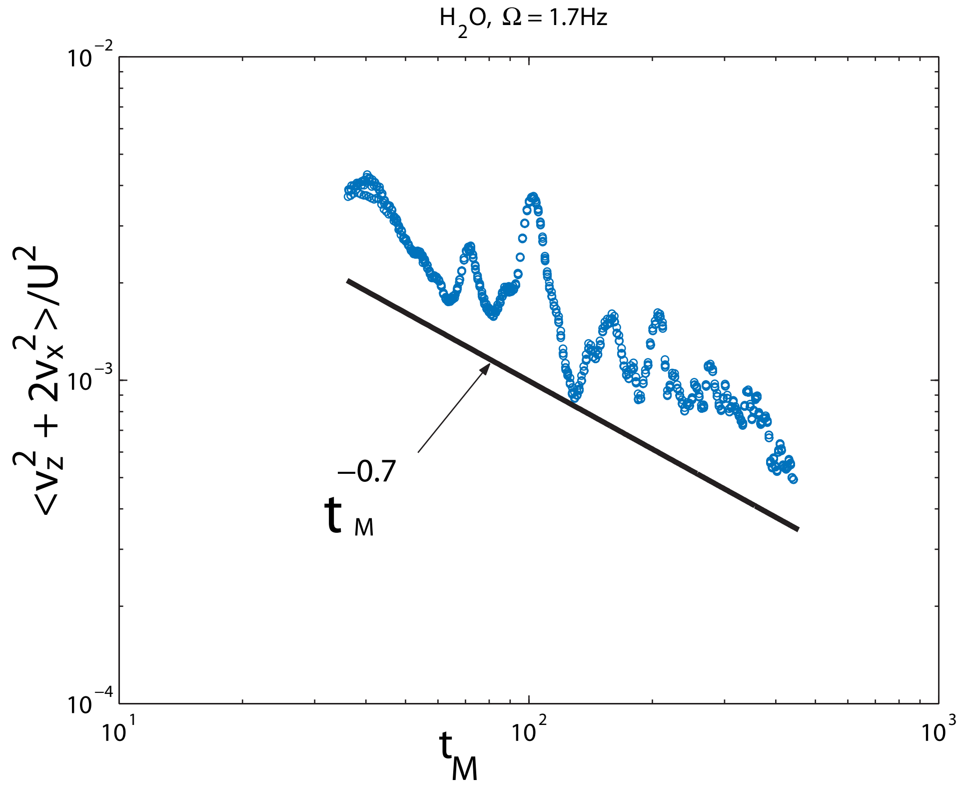


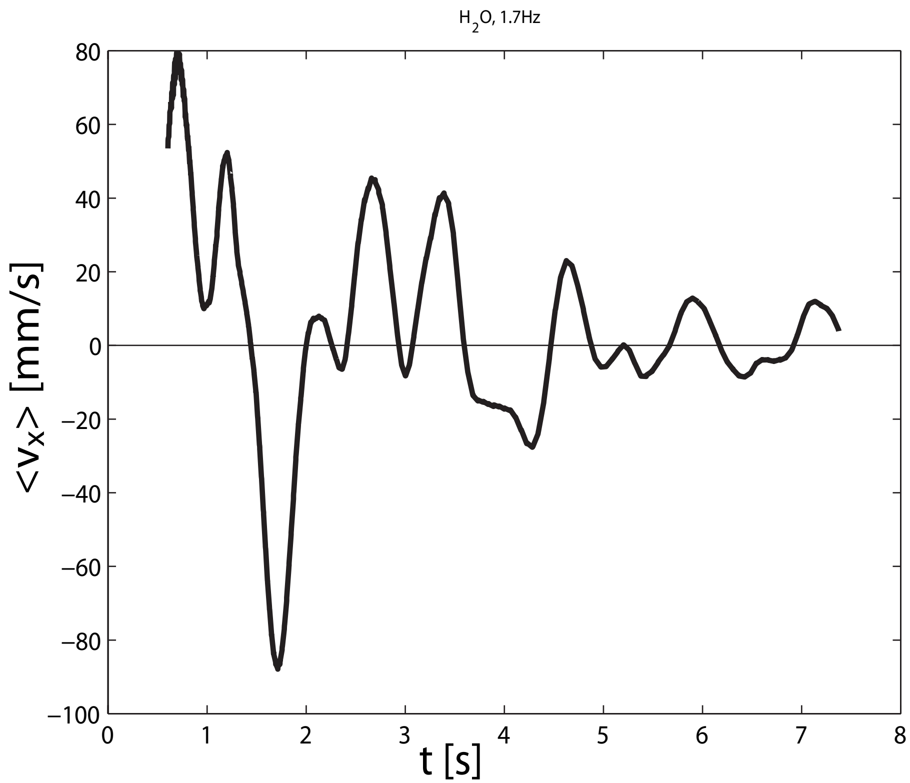
Decay of kinetic energy -- oscillations!

Water

Grid generated turbulence







$$\partial_t \vec{v} + 2\vec{\Omega} \times \vec{v} = -\frac{1}{\rho}\vec{\nabla} P$$

$$\partial_t \vec{\omega} = 2(\vec{\Omega} \cdot \vec{\nabla})\vec{v} = 2\Omega_o \partial_z \vec{v}$$

Plane wave solutions

$$\vec{v} = \vec{v}_o e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$\omega = \pm 2\Omega_o \frac{k_z}{k}$$

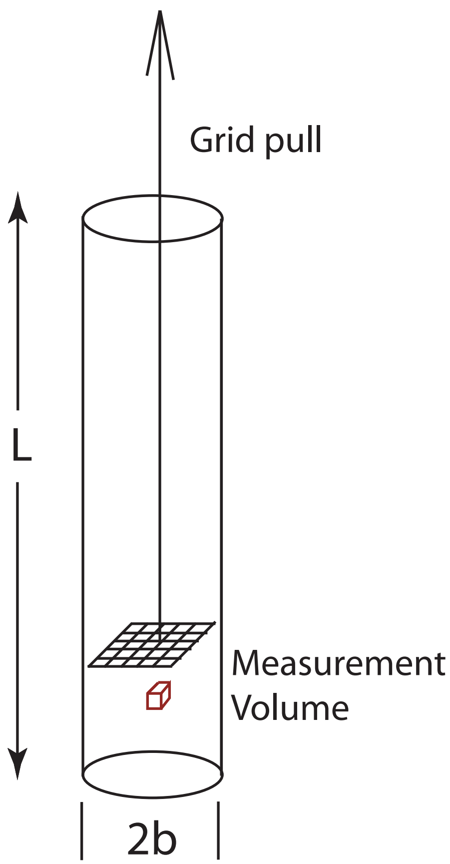
$$0 < |\omega| < 2\Omega_o$$

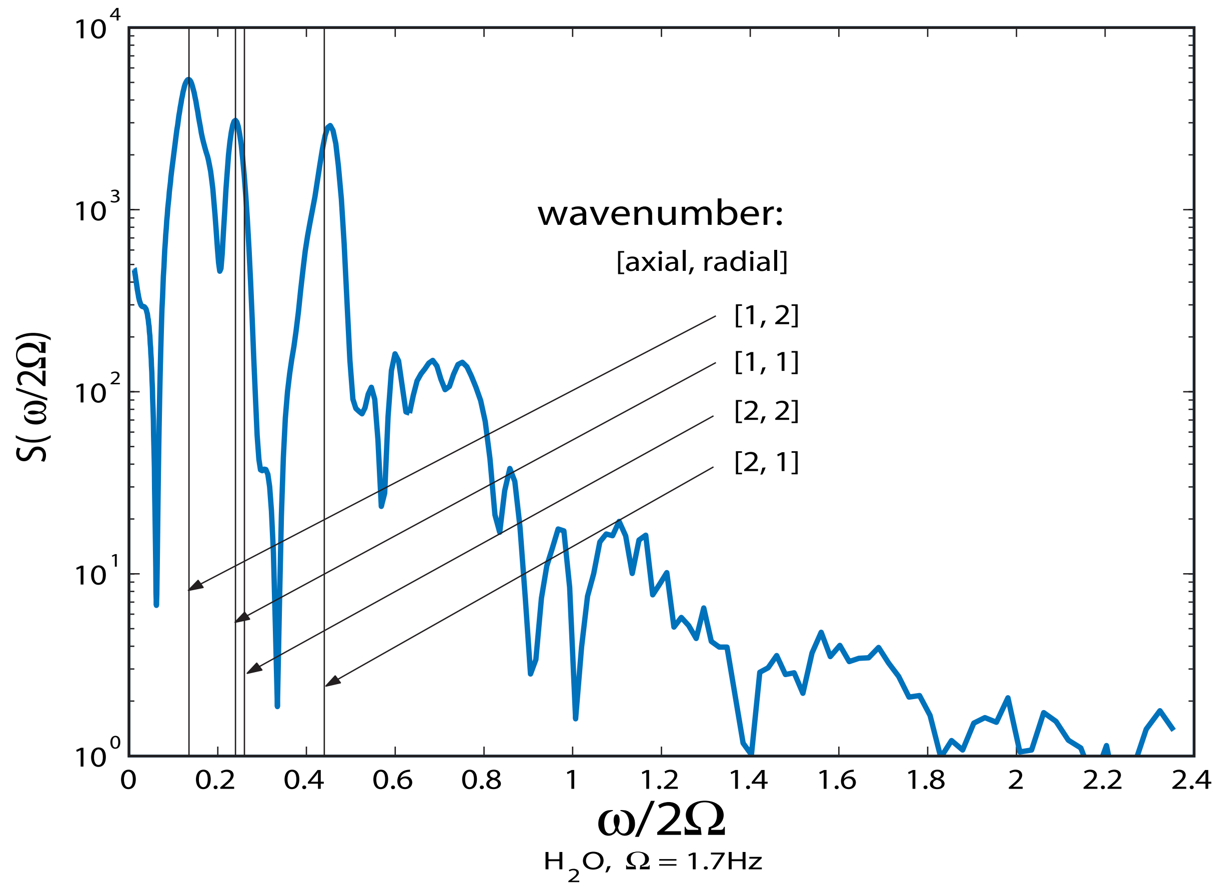
Modes of Containers

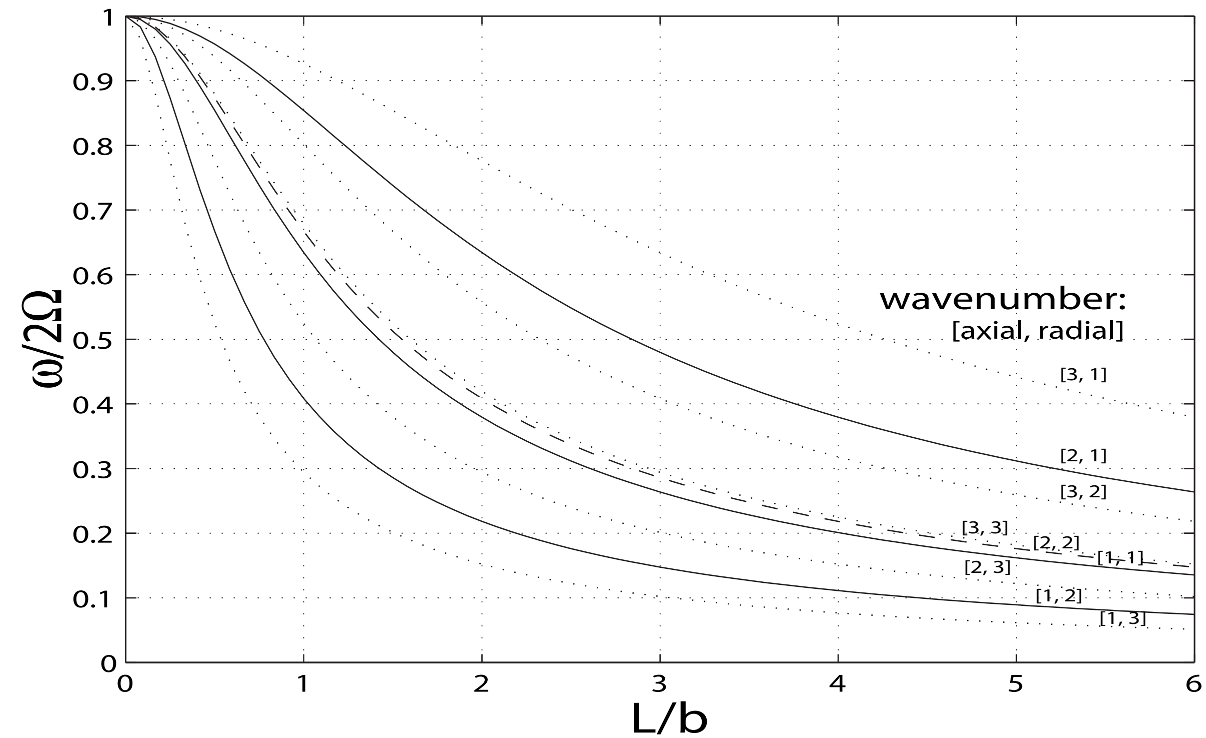
Frequency spectra of <v>

Water

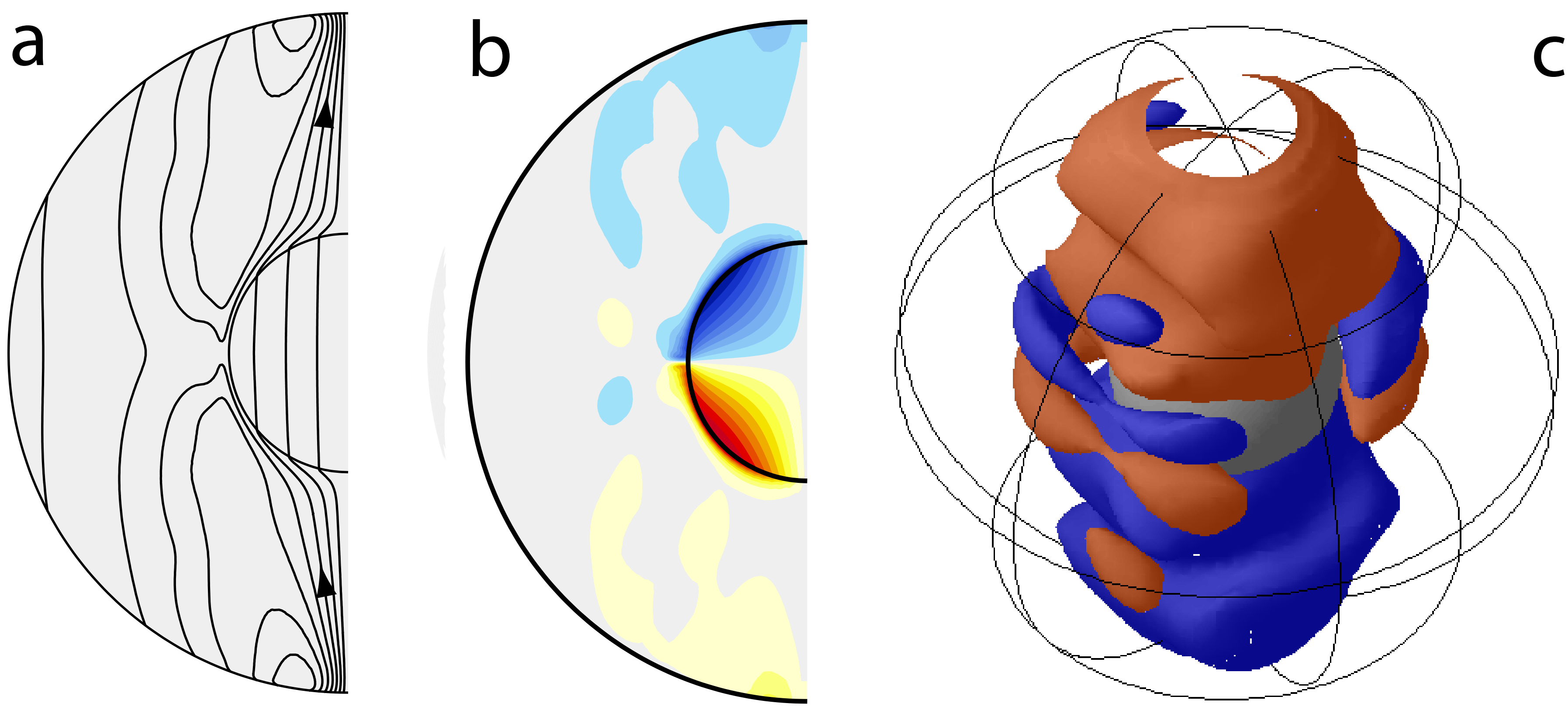
Grid generated turbulence





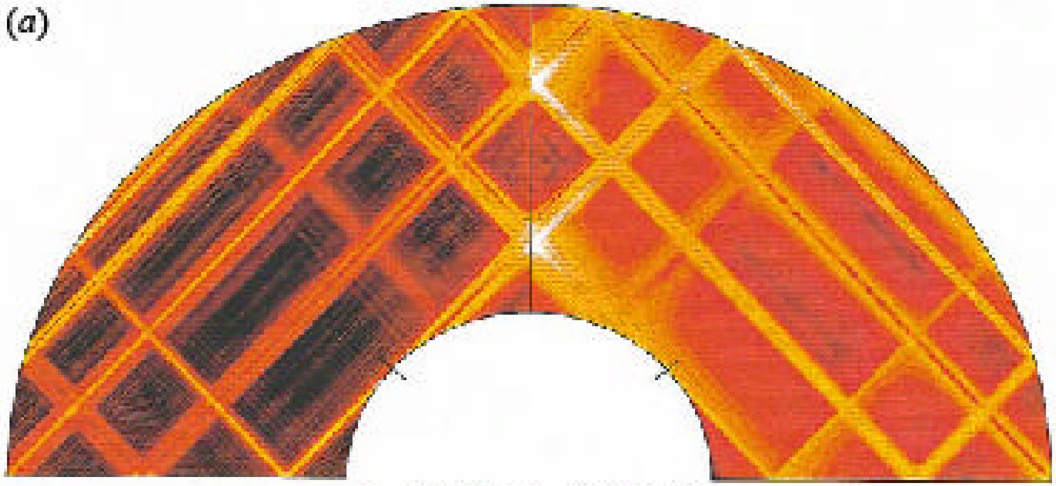


Simulations by Johannes Wicht
Dynamo action in Spherical Couette flow



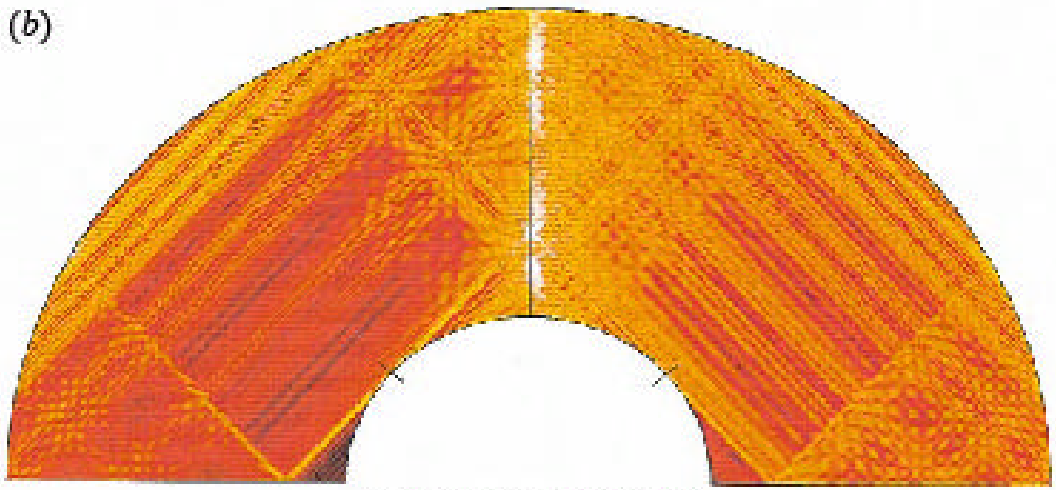
internal shocks

(a)



$\omega = 0.6598,\ \tau = -1.6 \times 10^{-4}$

(b)



$\omega = 0.6597,\ \tau = -6.6 \times 10^{-4}$

M. Rieutord and L. Valdettaro. Inertial Waves in a Rotating Spherical Shell. *Journal of Fluid Mechanics*, 341:77-99, 1997.

Equations of motion: known

$$\partial_t \vec{v} + (\vec{v} \bullet \vec{\nabla})\vec{v} + 2\vec{\Omega} \times \vec{v} = -\frac{1}{\rho}\vec{\nabla} P + \frac{1}{\rho\mu_o}(\vec{B} \bullet \vec{\nabla})\vec{B} + \nu\nabla^2\vec{v}$$

$$\partial_t \vec{B} + (\vec{v} \bullet \vec{\nabla})\vec{B} = (\vec{B} \bullet \vec{\nabla})\vec{v} + \eta\nabla^2\vec{B}$$

$$\vec{\nabla} \bullet \vec{v} = 0 \qquad \vec{\nabla} \bullet \vec{B} = 0$$

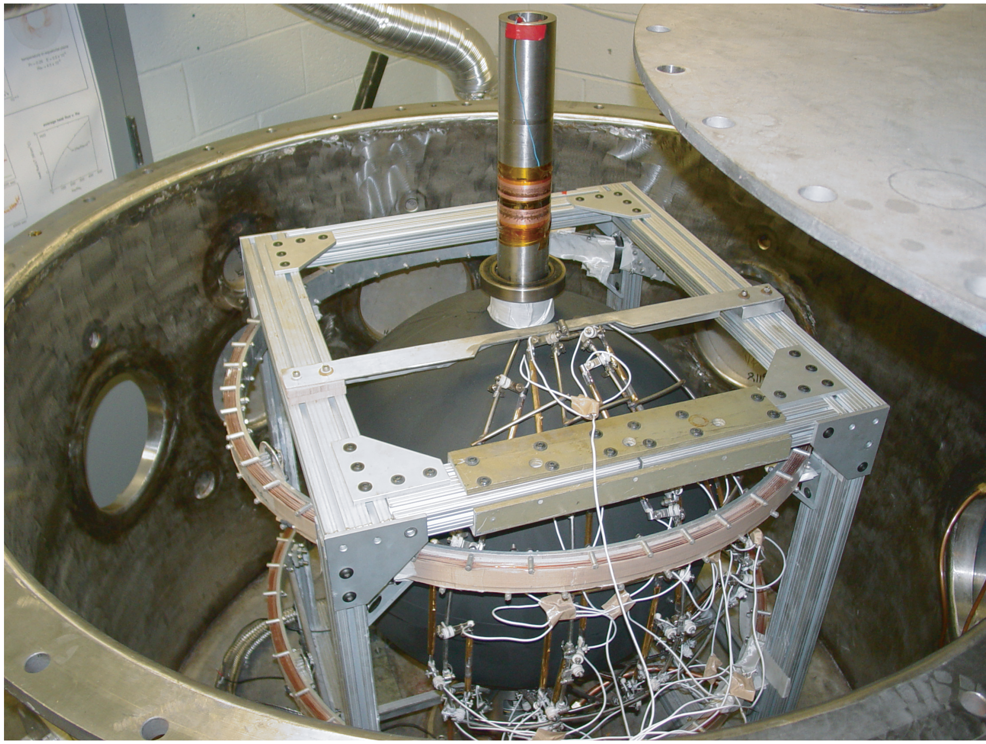
Key Parameters

$$R_m = \frac{UL}{\eta} = \frac{\text{magnetic field stretching}}{\text{resistive damping}} > 1$$

$$Pr_m = \frac{\nu}{\eta} = \frac{\text{momentum diffusivity}}{\text{magnetic diffusivity}} \sim o(10^{-5})$$

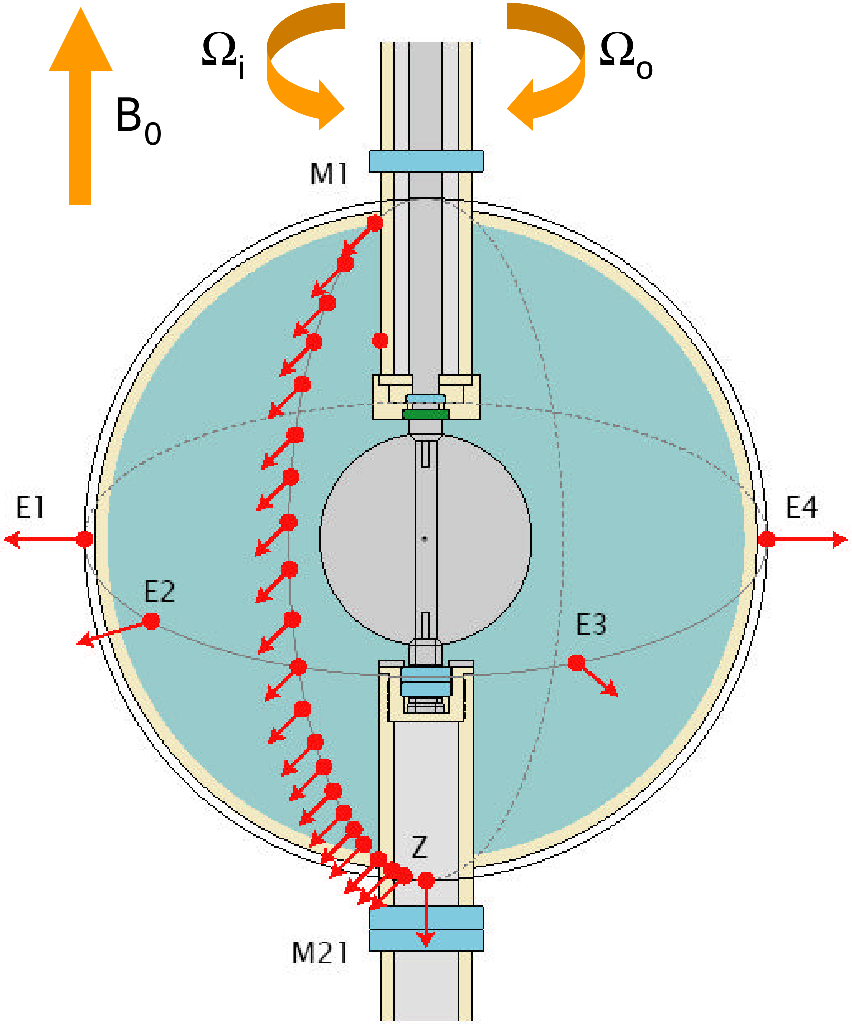
$$R = \frac{UL}{\nu} > o(10^5)$$

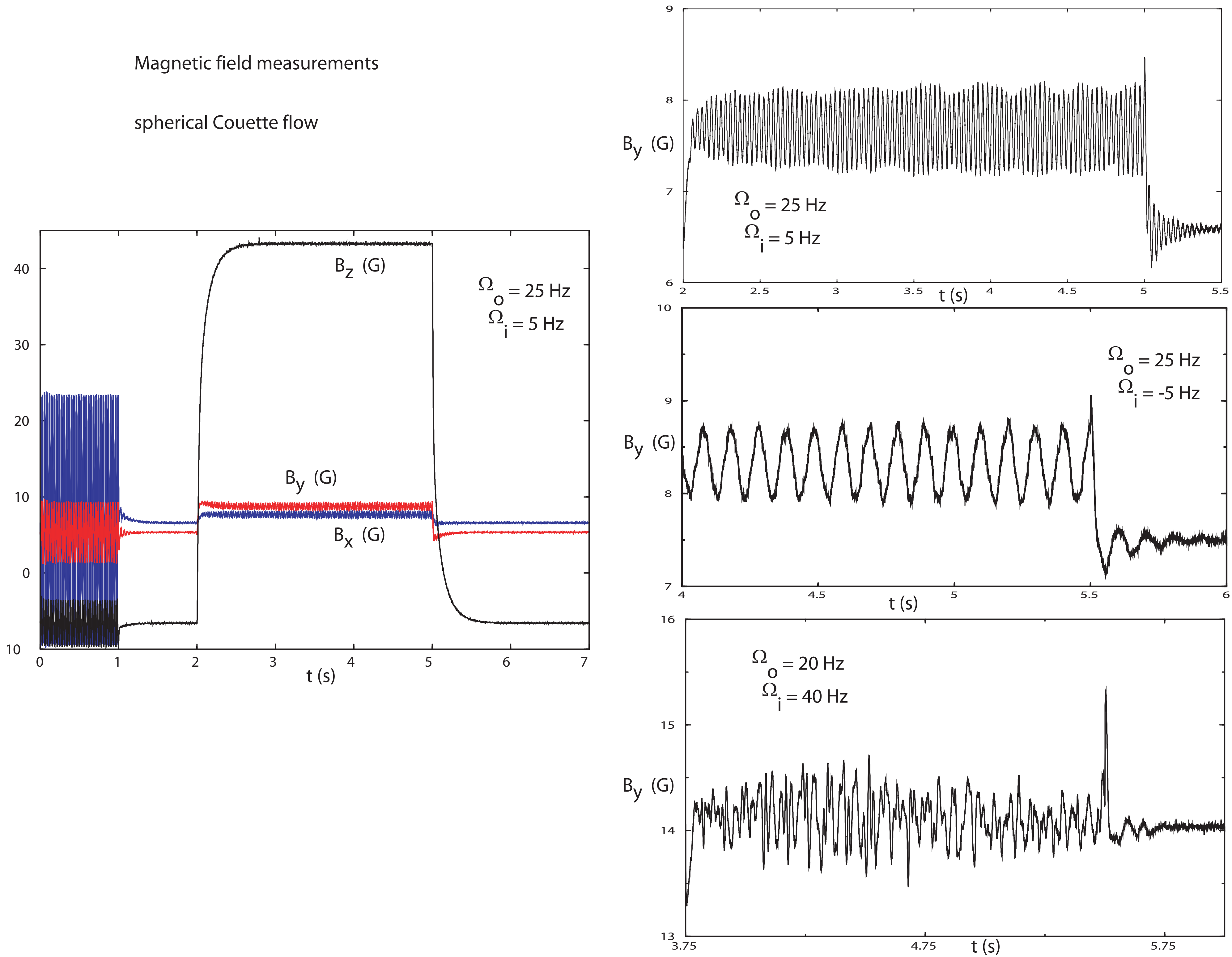
large R --> turbulence



$$S = \frac{B_0 L}{(\rho\mu_o)^{1/2}\eta} = \frac{\text{Alfven wave motion}}{\text{resistive damping}} > 1$$

60 cm experiment

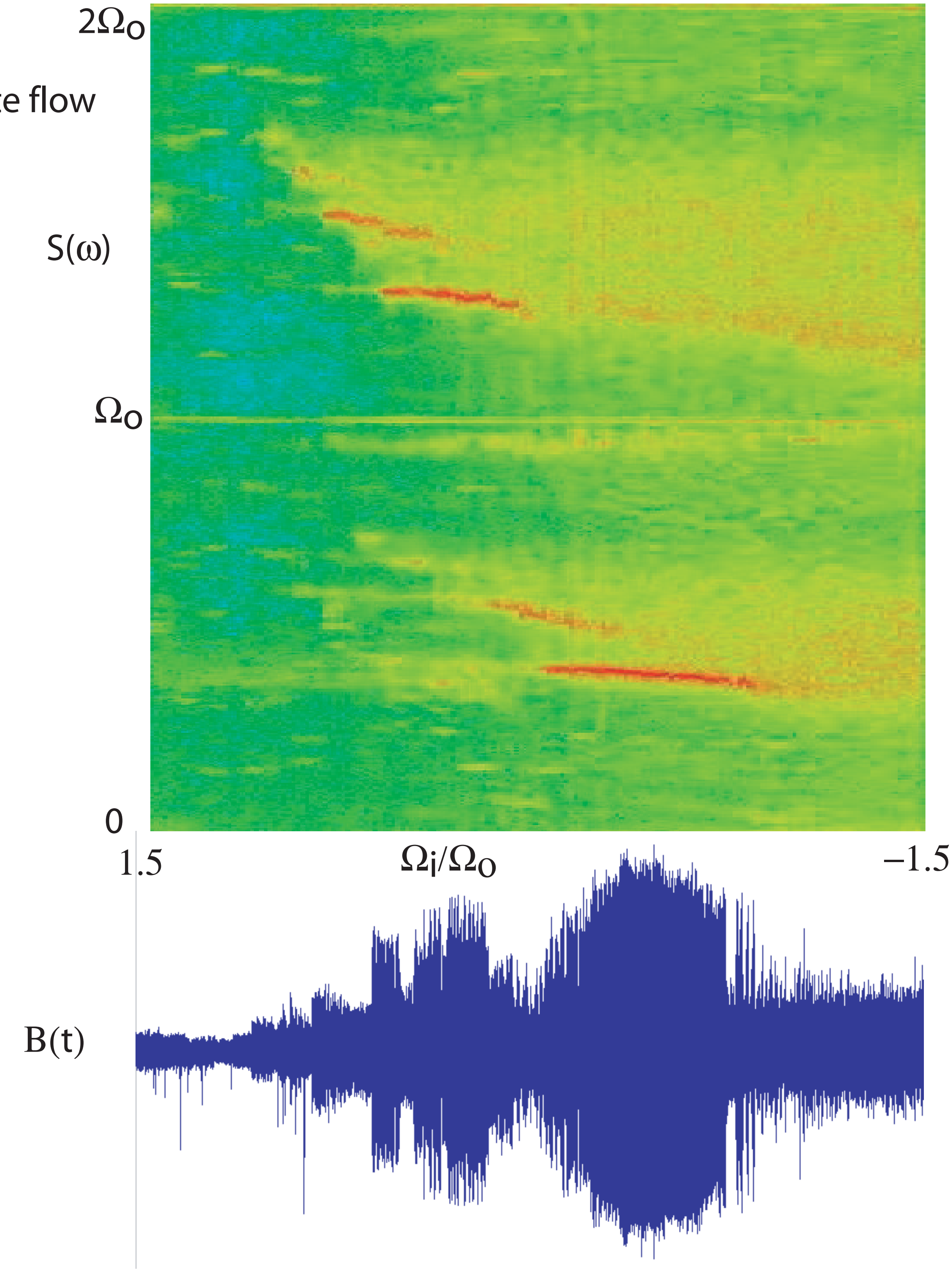




Magnetic field measurements
spherical Couette flow
B_z (G)
B_y (G)
B_x (G)
Ω_o = 25 Hz
Ω_i = 5 Hz
t (s)
B_y (G)
Ω_o = 25 Hz
Ω_i = 5 Hz
t (s)
B_y (G)
Ω_o = 25 Hz
Ω_i = -5 Hz
t (s)
B_y (G)
Ω_o = 20 Hz
Ω_i = 40 Hz
t (s)

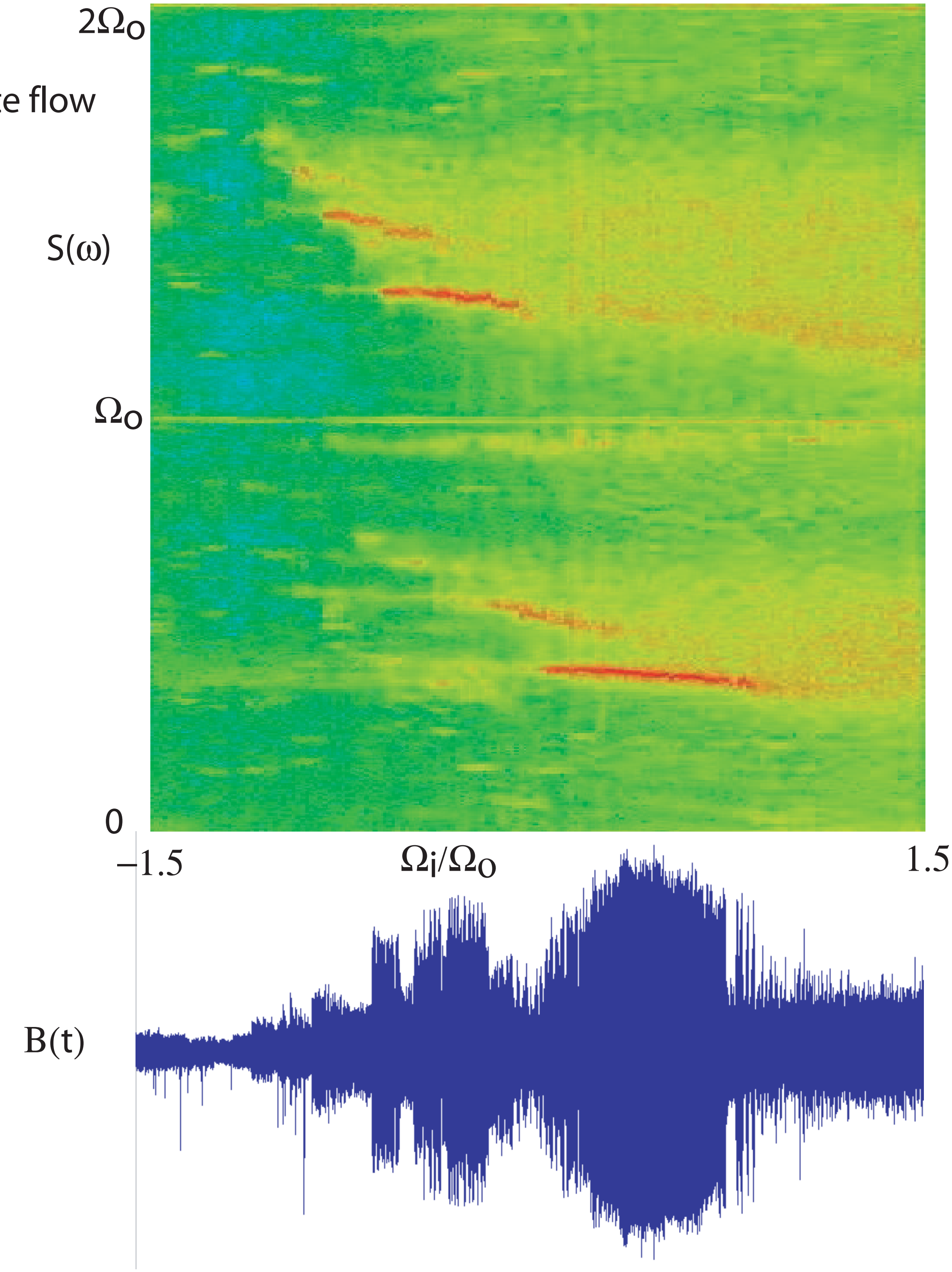
Modes in spherical Couette flow

$\Omega_o = 30$ Hz



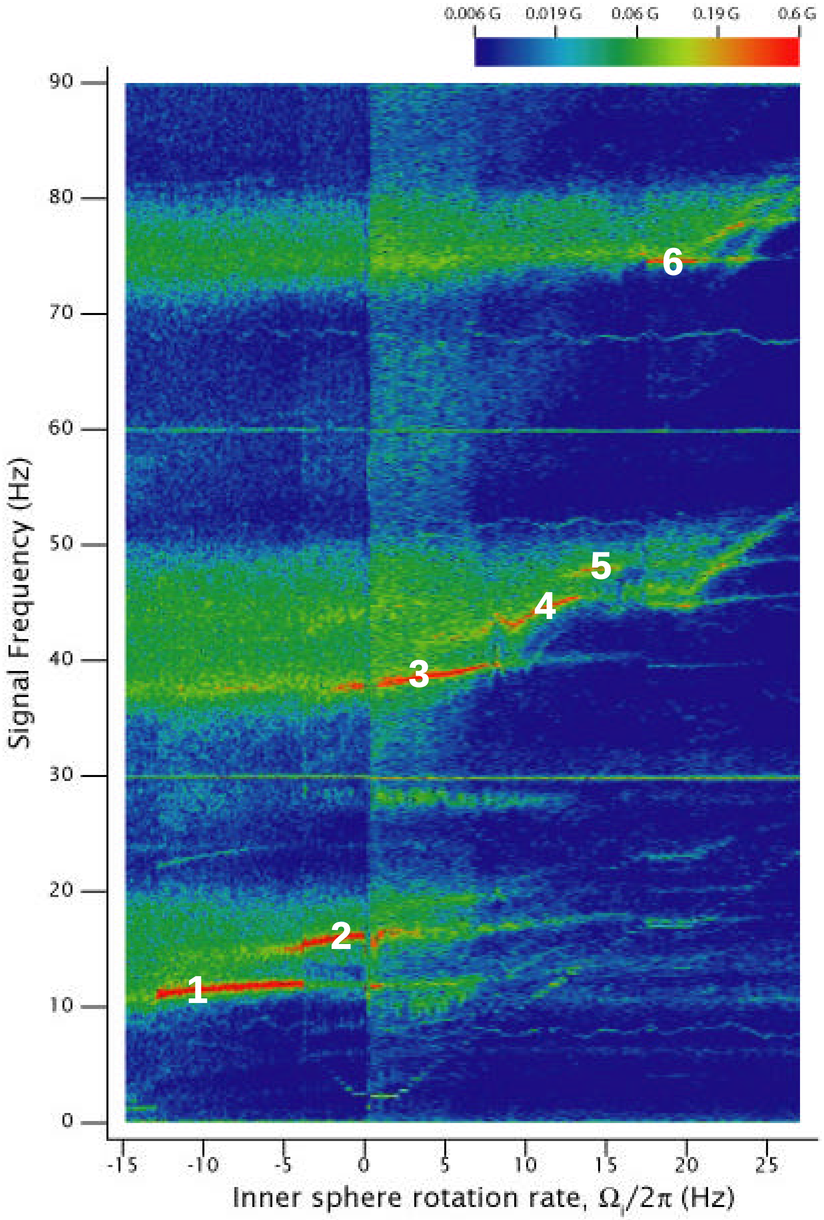
Modes in spherical Couette flow

$\Omega_O = 30$ Hz



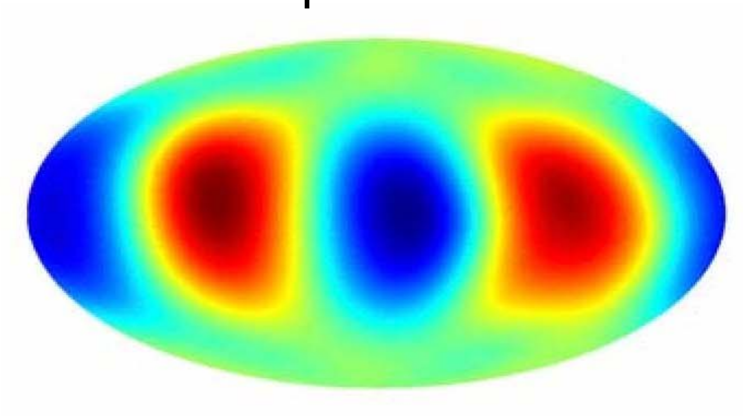
Y_2^2 dominant
Y_1^5 dominant
Y_1^3 dominant
Y_3^3 dominant
Y_2^4 dominant
Y_2^4 dominant
Signal Frequency (Hz)
Inner sphere rotation rate, Ω_i/2π (Hz)
0.006 G
0.019 G
0.06 G
0.19 G
0.6 G
B_φ (G)

Spectrogram

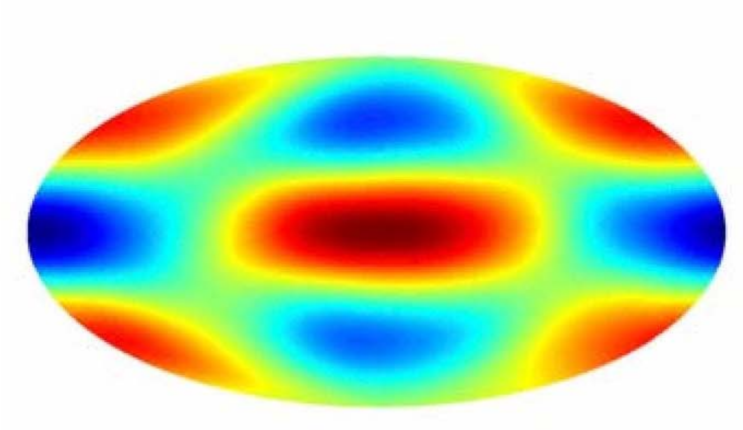


Induced magnetic field

Experiment

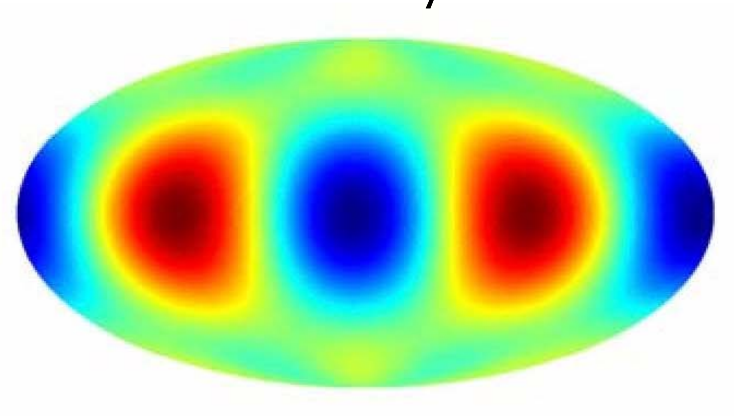


(a) $\omega_{lab}/\Omega_o = 1.30$, $\Omega_i = 5.7$ Hz

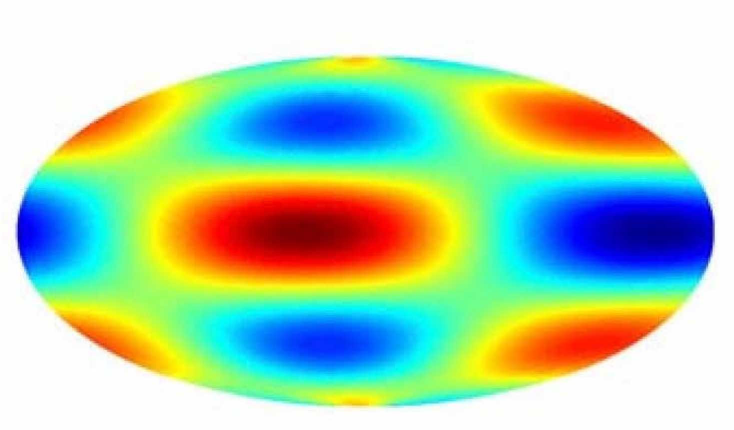


(c) $\omega_{lab}/\Omega_o = 0.39$, $\Omega_i = -12.2$ Hz

Theory



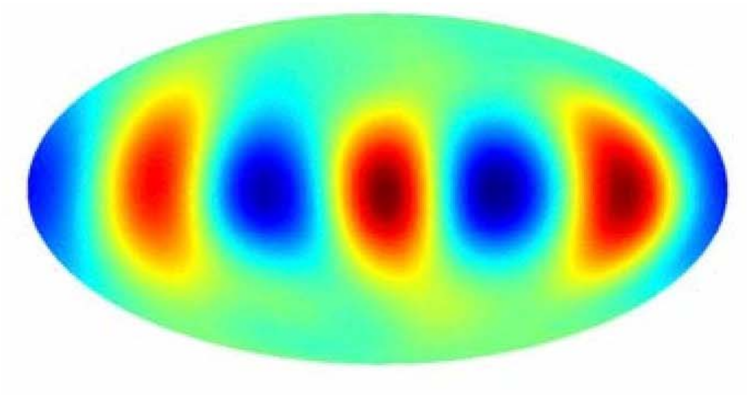
(b) $l_{mag} = 2$, $l = 3$, $m = 2$, $\omega/\Omega = 0.667$



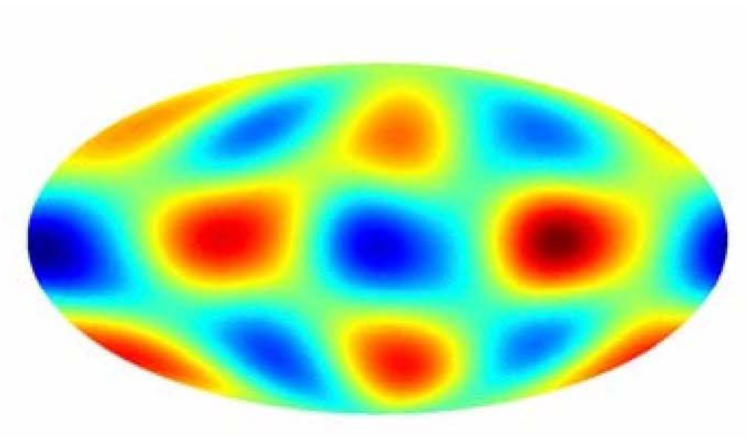
(d) $l_{mag} = 3$, $l = 4$, $m = 1$, $\omega/\Omega = 0.612$

Induced magnetic field

Experiment

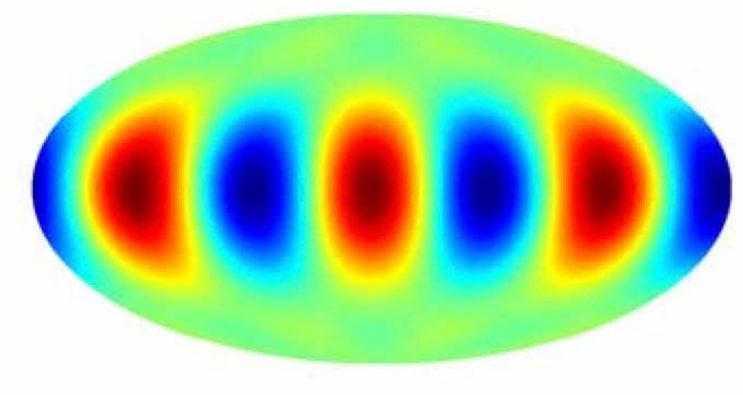


(e) $\omega_{lab}/\Omega_o = 2.50,\ \Omega_i = 16.8$ Hz

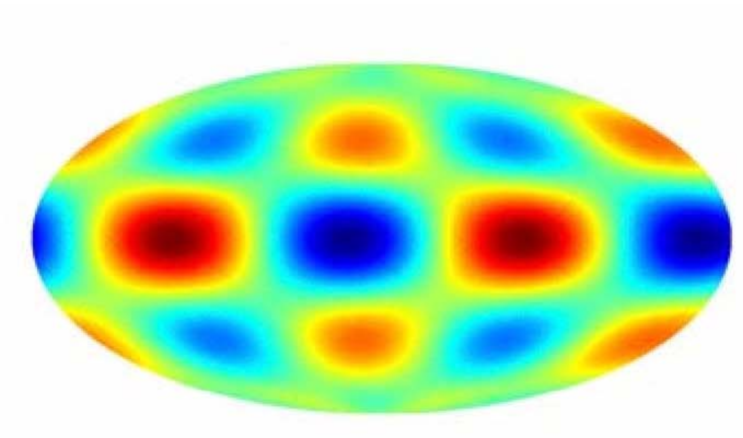


(g) $\omega_{lab}/\Omega_o = 1.51,\ \Omega_i = 12.0$ Hz

Theory



(f) $l_{mag} = 3,\ l = 4,\ m = 3,\ \omega/\Omega = 0.500$



(h) $l_{mag} = 4,\ l = 5,\ m = 2,\ \omega/\Omega = 0.467$

Conclusions

http://complex.umd.edu

B.W. Zeff, J. Fineberg, and D.P. Lathrop,
Nature 403, 401, (Jan. 27, 2000)

B.W. Zeff, D.D. Lanterman, R. McAllister,
R. Roy, E.J. Kostelich and D.P. Lathrop
Nature 421, 146, (Jan. 9, 2003)

D.P. Lathrop
arXiv:cond-mat/0311487

Bewley, Lathrop, and Sreenivasan
Nature 441, 588, (June 1, 2006)

Kelley, Triana, Zimmerman, and Lathrop
in preparation for GAFD