

Critical Thresholds in Eulerian Dynamics

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Abstract: We study the questions of global regularity vs. finite time breakdown in Eulerian dynamics, $u_t + u \cdot \nabla u = \nabla F$, which shows up in different contexts dictated by different modeling of F 's. To address these questions, we propose the notion Critical Threshold (CT), where a conditional finite time singularity depends on whether the initial configuration crosses an intrinsic, $\mathcal{O}(1)$ critical threshold. Our approach is based on a main new tool of spectral dynamics, where the eigenvalues, $\lambda := \lambda(\nabla u)$, and eigenpairs (ℓ, r) , are traced by the forced Riccati equation $\lambda_t + u \cdot \nabla \lambda + \lambda^2 = \langle \ell, D^2 F r \rangle$.

We shall outline three prototype cases.

We begin with the n -dimensional Restricted Euler equations, obtaining $[n/2]+1$ global invariants which precisely characterize the local topology at breakdown time. Next we introduce the corresponding n -dimensional Restricted Euler-Poisson (REP) system, identifying a set of $[n/2]$ global invariants, which yield (i) sufficient conditions for finite time breakdown, and (ii) a remarkable characterization of two-dimensional initial REP configurations with global smooth solutions. And finally, we show that rotation prevents finite-time breakdown. Our study reveals the dependence of the CT phenomenon on the initial spectral gap, $\lambda_2(0) - \lambda_1(0)$.