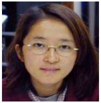
Extremely local MR representations:
L-CAMP

Youngmi Hur[1] & Amos Ron[2]

Workshop on sparse representations: UMD, May 2005



[1]Math, UW-Madison
[2]CS, UW-Madison

Wavelet and framelet constructions
History bits

- Pre-MRA: ad-hoc constructions (Meyer, Daubechies-Grossman-Meyer). (<87)

The common feature of all these innovations:

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The common feature of all these innovations: They do not scale with the dimension

Outline

The CAP methodologies

How local is “extremely local”?

L-CAMP: the algorithms
- Decomposition
- Reconstruction
- Complexity

L-CAMP: theory
- Wavelet-based characterizations of Besov spaces
- The key components in the L-CAMP performance analysis
- The performance chart
- An example: the mother of all local MR representations

L-CAMP: Extremely local MR constructions

Bird's view of the CAP methodologies

- CAP: universal construction of wavelets from any pair/triplet of lowpass filters.
  Complete performance analysis.

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  Performance: like CAP.
  Advantage: very local.

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- L-CAMP: a variant of CAMP. Available whenever the interpolatory filter in CAMP is tensor-product.
  Performance: like CAMP.
  Advantage: extremely local.

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L-CAMP: Extremely local MR constructions

The key elements of all these theories:

- CAP, CAMP, L-CAMP are computed by fast algorithms with linear complexity.

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L-CAMP: Extremely local MR constructions

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Test problem. Build an “extremely local” system that analyses correctly and completely C^1-functions in high dimension.

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Test problem. Build an “extremely local” system that analyses correctly and completely C^1-functions in high dimension.

Quantifying “local”: the number of linear functionals whose support contain a given generic point $x \in \mathbb{R}^n$.

Extremely local, first try

Ave, Caesar! Morituri ti Salutamus!

Goal: analyse C^1-functions in $\mathbb{R}^{10}$.

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Hint for the confused: that is not local at all!!!

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Meaning:[3]

What takes $3/5$ wavelets 50 years,
is a one-minute job for splines

[3] Assuming linear complexity with constants that depend on volume

Extremely local, a third try

CAP[4]: *The Empire Strikes Back*[5]

- The 11Dir box spline is used to construct MR.
- The wavelets are only implicit
- We get 1024 wavelets with average volume of support ≈ 30

Updated table:

[4] Youngmi Hur + AR, 2005,
`ftp://ftp.cs.wisc.edu/Approx/huron.pdf`

[5]

Just like moons and like suns,
With the certainty of tides,
Just like hopes springing high,
Still I'll rise – M. Wavelet

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Updated table:

Not bad, but we'd better find a *sombrero*

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Extremely local, a fourth try

CAMP[6]: *With colors bright, the sun did rise*

- The 11Dir box spline is interpolatory, hence CAMP is available
- The wavelets, again, are only implicit
- We get, again, 1024 wavelets with average volume of support a secret.

6

In desperation did I pray
For just a single morning ray,
For sun to pierce this darkest night,
And, with this death, bring forth new light

Extremely local, the final try

L-CAMP: “... In the Land of Mordor where the Shadows lie”

The breakthrough is based on separating between the inversion

The table:

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Q. Are we performing as good as piecewise-linears?

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▸ The table:

Q. Are we performing as good as piecewise-linears?

A.

- **Good news:** C^1 *are* covered (not “barely” covered). In fact, the representation analyses smoothness up to $C^{1.4}$.
- Bad news: unlike piecewise-linears, we miss the Hardy space H^1 in the performance analysis

L-CAMP: The algorithms

Decomposition

Step I: choose three lowpass filters

$$\begin{aligned} h_c &:= 2^{-n} \sum_{\nu \in \{0,1\}^n} \delta_\nu. \\ h_e &:= n - \text{dimensional (to be discussed)} \\ h &:= 1 - \text{D, supported on the odd integers} \end{aligned}$$

L-CAMP: The algorithms

Decomposition

Step I: choose three lowpass filters

Step II: build the MRA

$\downarrow$ is downsampling:

$$y_{\downarrow}(k) \;=\; y(2k), \quad k \in \mathbb{Z}$$

$(y_j)_{j=-\infty}^{\infty} \subset \mathbb{C}^{\mathbb{Z}^n}$ s.t:

$$y_{j-1} = Cy_j := (h_c * y_j)_{\downarrow}, \quad \forall j.$$

L-CAMP: The algorithms

Decomposition

Step I: choose three lowpass filters

Step II: build the MRA

Step III: extract detail coefficients:

(1) For $k \in 2\mathbb{Z}^n$,

$$d_j(k) := y_j(k) - (h_e * y_{j-1})(\frac{k}{2}).$$

(2) For $\nu \in \{0, 1\}^n$, and $k \in \nu + 2\mathbb{Z}^n$,

$$d_j(k) = y_j(k) - (h_{J(\nu)} * y_j)(k).$$

$h_{J(\nu)} =?$

L-CAMP: The algorithms

Decomposition

Step I: choose three lowpass filters

Examples of h:

$$h = [\mathbf{0}, 1], \quad h = [\frac{1}{2}, \mathbf{0}, \frac{1}{2}], \quad h = \frac{1}{16} \times [-1, 0, 9, \mathbf{0}, 9, 0, -1].$$

back to performance

Step II: build the MRA

Step III: extract detail coefficients:

L-CAMP: The algorithms

Reconstruction

Step I: for $k \in 2\mathbb{Z}^n$,

$$y_j(k) := d_j(k) + (h_e * y_{j-1})(k/2).$$

L-CAMP: The algorithms

Reconstruction

Step I: for $k \in 2\mathbb{Z}^n$,

$$y_j(k) := d_j(k) + (h_e * y_{j-1})(k/2).$$

Step II: iteratively, by suitably ordering $\{0,1\}^n \backslash 0$:

$$y_j(k) = d_j(k) + (h_{J(\nu)} * y_j)(k).$$

L-CAMP: The algorithms

Complexity

Denote: h_e is A-tap, h is B-tap

---

[7] per one complete cycle of decom-recon

L-CAMP: The algorithms
Complexity

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Then:
Decomposition requires for 2^n detail coefficients:
$2^n + A + 1 + (B + 1) \times (2^n - 1)$.

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Average # of operations per one detail coefficient [7]

$$2B + 3 + 2^{1-n}(A+1).$$

[7] per one complete cycle of decom-recon

L-CAMP: Theory

Introduction: wavelet-based characterizations of function spaces

Let $\psi \in L_2(\mathbb{R}^n) \cap L_1(\mathbb{R}^n)$ s.t. $\int \psi(t)dt = 0$

Wavelet system $X(\Psi)$ is

$$X(\Psi) := \left\{ \psi_{j,k} = 2^{jn/2}\psi\left(2^j \cdot - k\right) : \ \psi \in \Psi, \ j \in \mathbb{Z}, \ k \in \mathbb{Z}^n. \right\}.$$

The Besov space B^s_{pp} ($s \in \mathbb{R}, 0 < p < \infty$)

Let $\varphi \in \mathcal{S}$ satisfy

$$\begin{aligned} \operatorname{supp} \widehat{\varphi} \subset \{1/2 \le |\xi| \le 2\}, & \\ |\widehat{\varphi}(\xi)| \ge c > 0, & \qquad 3/5 \le |\xi| \le 5/3, \\ \textstyle\sum_{j \in \mathbb{Z}} |\hat{\varphi}(2^{-j}\xi)|^2 = 1, & \qquad \xi \in \mathbb{R} \setminus \{0\}. \end{aligned}$$

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The function space B^s_{pp} is the set of all $f \in \mathcal{S}'/\mathcal{P}$ s.t.

$$\|f\|_{B^s_{pp}} := \left\| \left(\sum_{j\in\mathbb{Z}} (2^{js}|\varphi_j * f|)^p \right) \right\|_{L_p} < \infty, \quad \varphi_j := 2^{jn}\varphi(2^j\cdot).$$

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- $B^0_{22} \cap B^m_{22} \approx W^m_2$ (the Sobolev space).

L-CAMP: Theory

Characterization of Besov spaces using wavelets

Theorem (Meyer, Frazier-Jawerth, 198x)

$n > \max\{s, -s, n(\frac{1}{p} - 1) - s\}$, integer.
$X(\Psi)$ is orthonormal wavelet, and:

$$\psi \in C_c^m, \qquad \int t^\alpha \psi(t) dt = 0, \quad \forall 0 \leq |\alpha| \leq m - 1.$$

Then we have

$$\|f\|_{B_{pp}^s} \approx \|Q_\psi^s f\|_{L_p},$$

where

$$Q_\psi^s f := \left(\sum_{\psi,j,k} \left| \langle f, \psi_{j,k} \rangle \, 2^{js} \, \chi_{j,k} \right|^p \right)^{1/p},$$

L-CAMP: Theory

framelet-based characterizations of function spaces

Theorem (Kyriazis, Nielsen)

$s \in \mathbb{R}$, $m > \max\{s, -s, n(\frac{1}{p} - 1) - s\}$, integer.
$X(\Psi)$ is frame and:

$$\Psi \subset C_c^m, \qquad \int t^\alpha \psi(t) dt = 0, \quad \forall 0 \leq |\alpha| \leq m-1, \forall \psi \in \Psi \quad (1)$$

Then we have $\|f\|_{B^s_{pp}} \approx \|Q^s_\Psi f\|_{L_p}$, where

$$Q^s_\Psi f := \left(\sum_{\psi \in \Psi, j,k \in \mathbb{Z}} \left| \left\langle f, \psi_{j,k} \right\rangle 2^{js} \chi_{j,k} \right|^p \right)^{1/p} .$$

L-CAMP: Performance analysis

The key components

- **The accuracy of the univariate filter h:**

$$h * P = P, \quad \forall \text{ univariate polynomial } P \text{ of degree} < s_1$$

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- **The accuracy of the pair (h_c, h_e):**

$$(h_{e\uparrow} * h_c) * P = P, \quad \forall \text{ multivariate polynomial } P \text{ of degree } < s_2$$

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- **The accuracy of the univariate filter h:**

$$h * P = P, \quad \forall \text{ univariate polynomial } P \text{ of degree} < s_1$$

- **The accuracy of the pair (h_c, h_e):**

$$(h_{e\uparrow} * h_c) * P = P, \quad \forall \text{ multivariate polynomial } P \text{ of degree} < s_2$$

- **The smoothness class of the refinable function ϕ^d** whose mask is

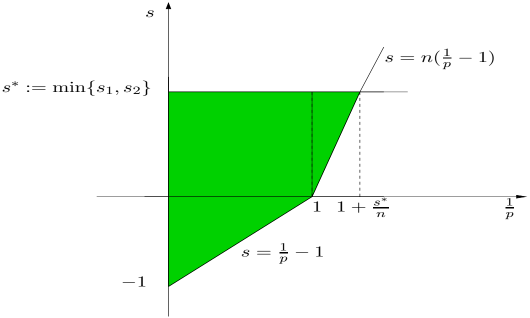
$$\widehat{h}_e(2\cdot)\widehat{h}_{tensor},$$

with h_{tensor} the n-dimensional tensor-product of h.

L-CAMP: Performance analysis

Jackson-type performance chart

performance chart



L-CAMP: Performance analysis

An example: the mother of all local MR representations

$$\begin{aligned} h \quad &:= [\frac{1}{2}, \mathbf{0}, \frac{1}{2}], \quad \text{2-tap}, \\ \widehat{h}_e(\omega) \quad &:= \frac{3}{4} + \frac{1}{4} e^{i\mathbf{1}\cdot\omega}, \quad \text{2-tap}. \end{aligned}$$

- **The accuracy of the univariate filter h:** $s_1 = 2$.
- **The accuracy of the pair (h_c, h_e):** $s_2 = 2$.
- **The smoothness class of the refinable function ϕ^d whose mask is $\widehat{h}_e(2\cdot)\widehat{h}_{tensor}$:** $s_3 > 1$ ($s_3 = 1.4$?).

Average # of operations: $7 + 3 \cdot 2^{1-n}$.

Total volume of the wavelets' support: < 5.

Finalement,
c'est fini!

The 3-tap wavelet in the 5/3 system

back

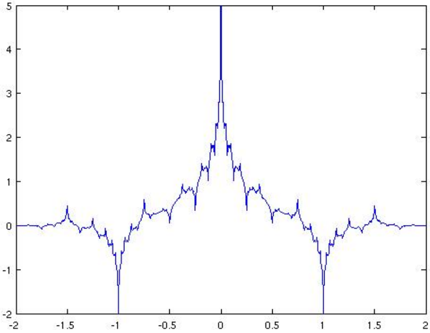


Table 1

wavelets	splines
$275,000$	$.01075$

back

Table 2

wavelets	splines	CAP
$275,000$	$.01075$	30

back

Table 3

wavelets	splines	CAP	L−CAMP
275, 000	.01075	30	.004889

back

The shape of things to come...

The breakthrough is based on separating between the inversion (=reconstruction) and the dual system.

back

1

Three Rings for the elven kings, under the sky
Seven for the dwarf lords, in their halls of stone
Nine for mortal men, doomed to die
and one for the Dark Lord, on his dark throne
in the land of Mordor, where the shadows lie

2

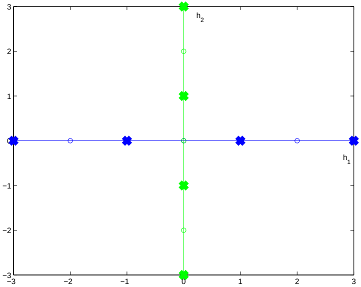
One Ring to rule them all*, One Ring to find them*
One Ring to bring them all, and in the darkness bind them
in the land of Mordor, where the shadows lie

The shape of things to come...

The Ring

back

Orienting the univariate filter



back